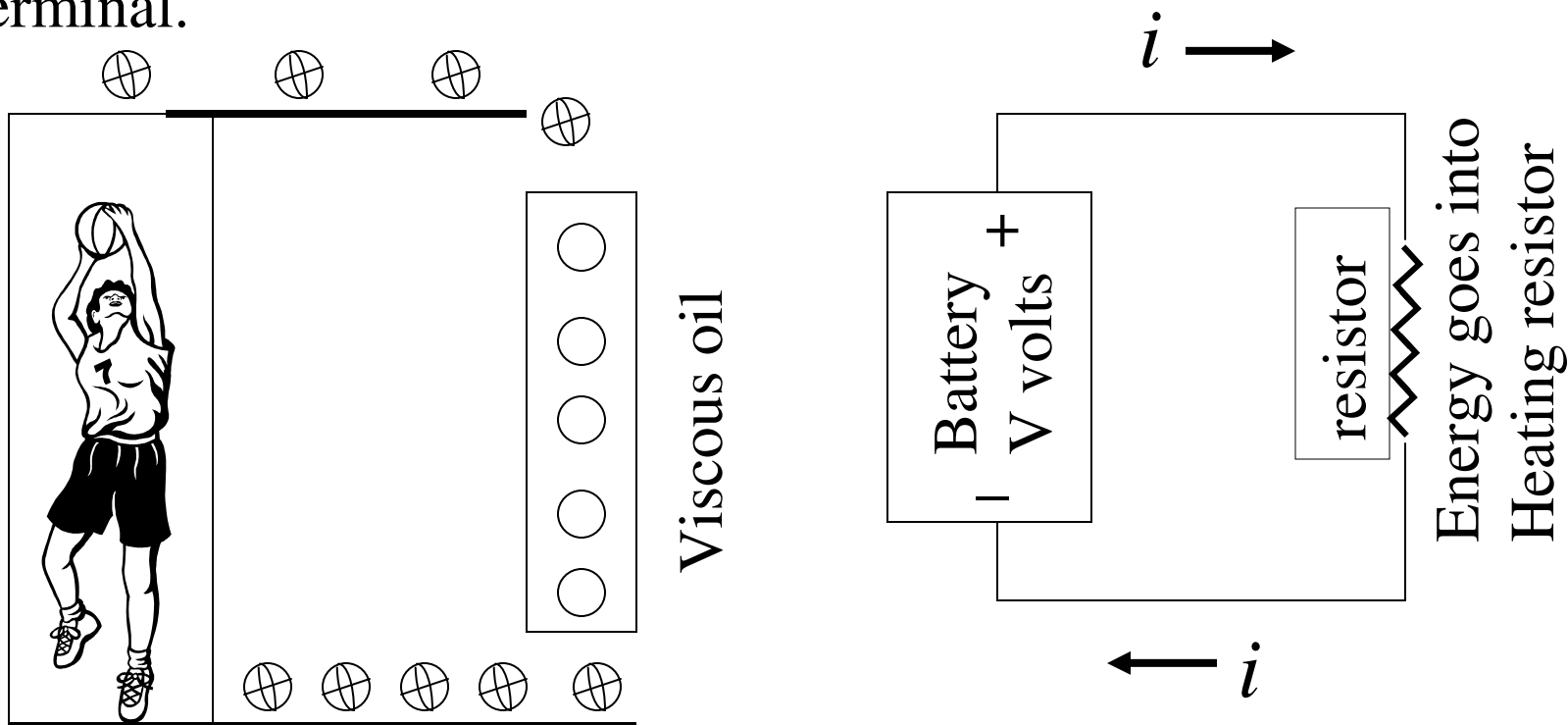


Lecture 9 – Phys 272  
Currents, EMF (voltage) Sources &  
Power

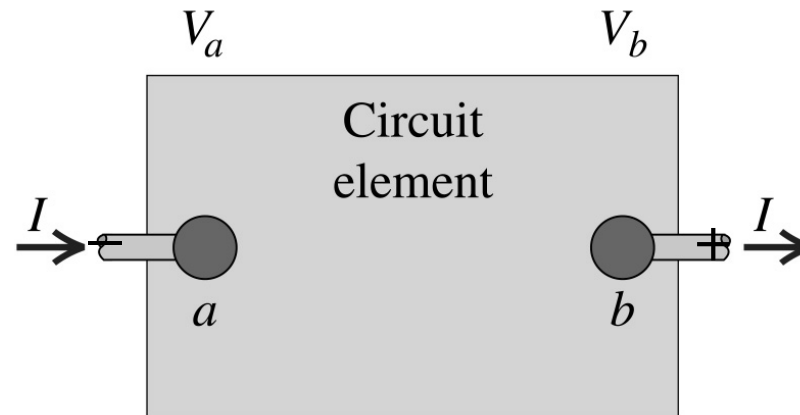
# Electromotive Force and Circuits

Here we discuss “sources of electromotive force”, such as batteries, electric generators and solar cells, and we consider how they behave in a closed circuit. Below is an analogy between basketball and current in a closed circuit. The player does work to move the ball up and the ball loses energy in the viscous oil giving off heat. The battery does work to move  $q$  from  $-$  to  $+$  terminals, gaining energy  $qV$ . The charge  $q$  moves through wire to resistor and loses energy in the resistor and goes to the  $-$  terminal.

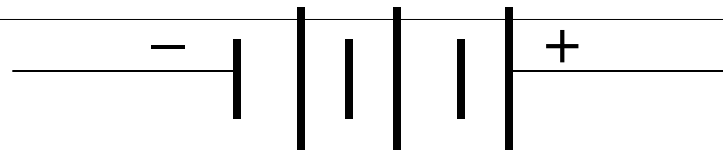


## Electromotive Force (EMF)

A battery is a device that keeps a & b terminals at a fixed potential difference and will move a positive charge or current from the a to the b terminals by some process such as electrolysis (storage battery) or the photoelectric effect (solar cells). This “force” is called the *electromotive force* or EMF or script  $\mathcal{E}$ .



The battery circuit diagram is given by perpendicular lines, with + terminal with the bigger line.

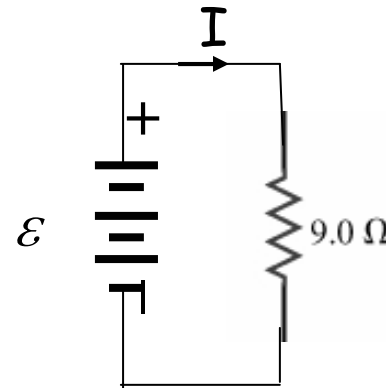


## Idealized Battery

The ideal battery with potential  $\varepsilon$  has no internal resistance and is represented by



V across battery is  $\varepsilon$  no matter what current is drawn! Not very physical.



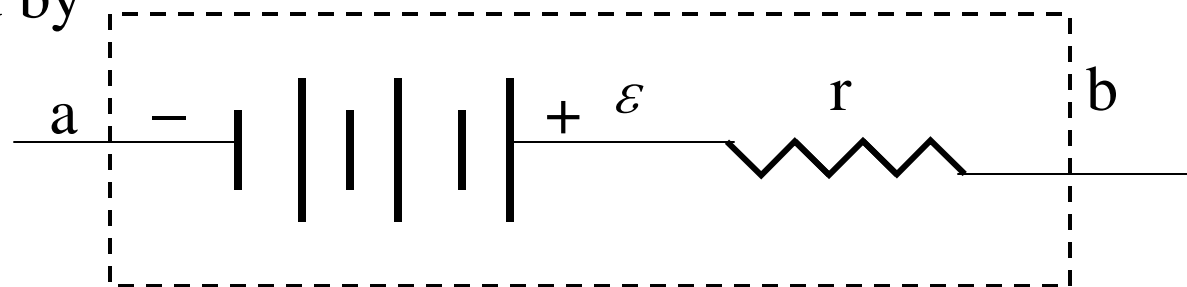
$$V_B = V_R$$

$$\varepsilon = IR$$

$$I = \frac{\varepsilon}{R}$$

## "Real Battery"

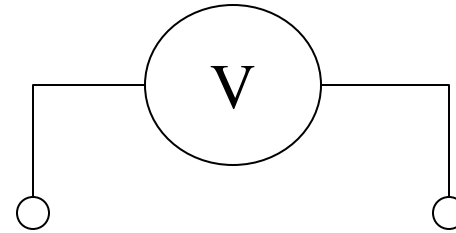
The real battery with potential  $\varepsilon$  has a small internal resistance  $r$  and is represented by



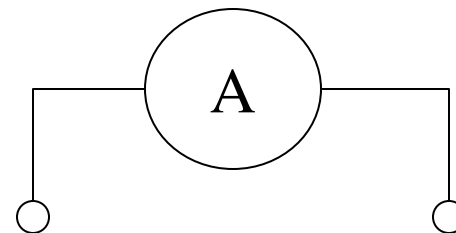
Voltage, between a & b, is slightly less than  $\varepsilon$

# Electrical Meters

Voltmeter, measures voltage and has large internal resistance. An ideal voltmeter has infinite resistance and does not conduct any current.



Ammeter, measures current and has very small internal resistance. An ideal ammeter has zero resistance and behaves like a conducting wire.



How should we connect these meters in circuits ?

Real Battery with 12volts and an internal resistance of 2 ohms in a circuit with a resistor of 4 ohms

$$V_{AB} = V_{A'B'}$$

$$\mathcal{E} - Ir = IR$$

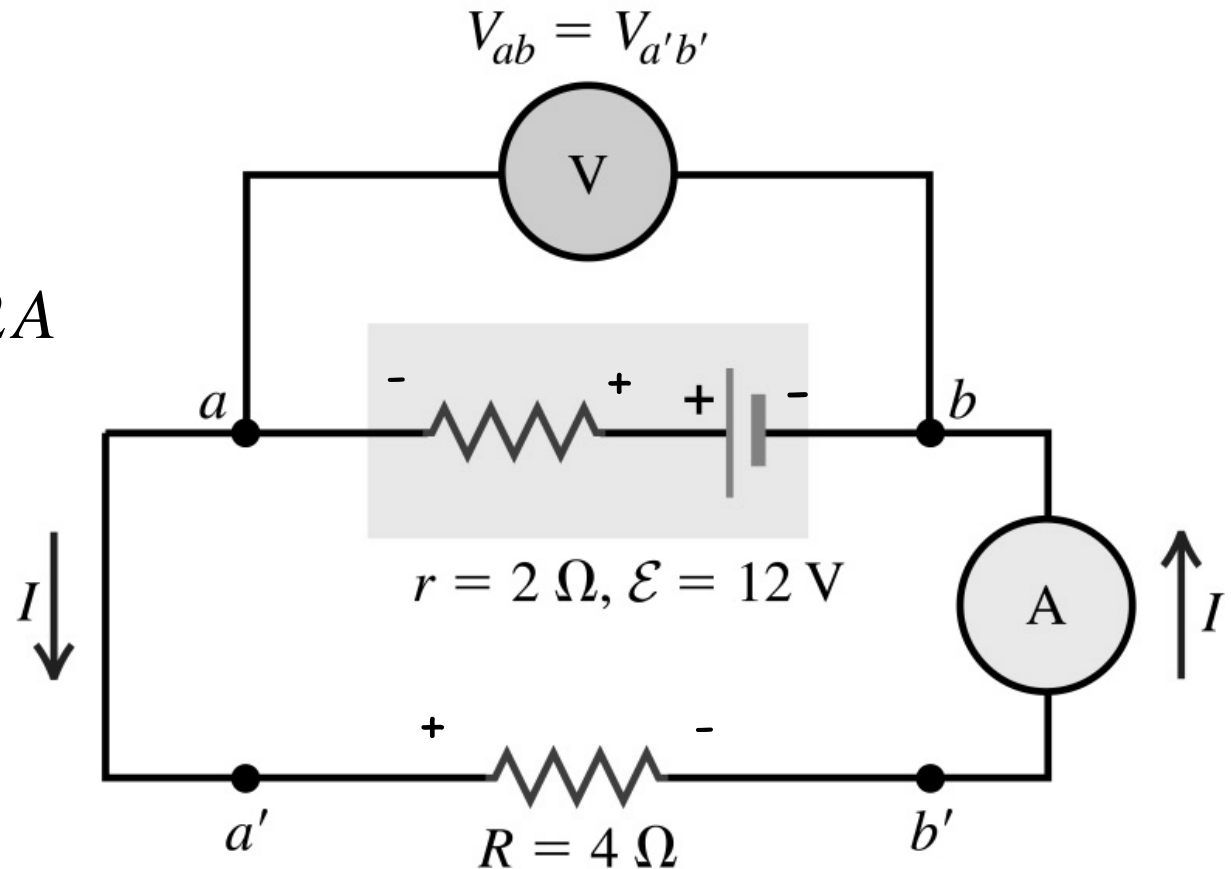
$$I(R + r) = \mathcal{E}$$

$$I = \frac{\mathcal{E}}{R + r} = \frac{12V}{6\Omega} = 2A$$

Voltage drop across 2 Ohms is 4 volts

Voltage drop across 4 Ohms is 8 volts

$$V_{ab} = V_{a'b'} = 8 \text{ volts}$$



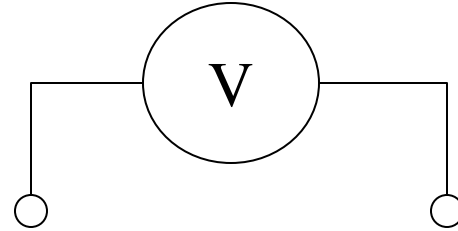
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Terminal potential difference  $V_{AB} = 12 - 4 = 8\text{V}$

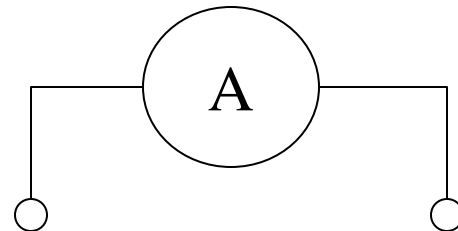
# Electrical Meters

How should we connect these meters in circuits ?

In parallel

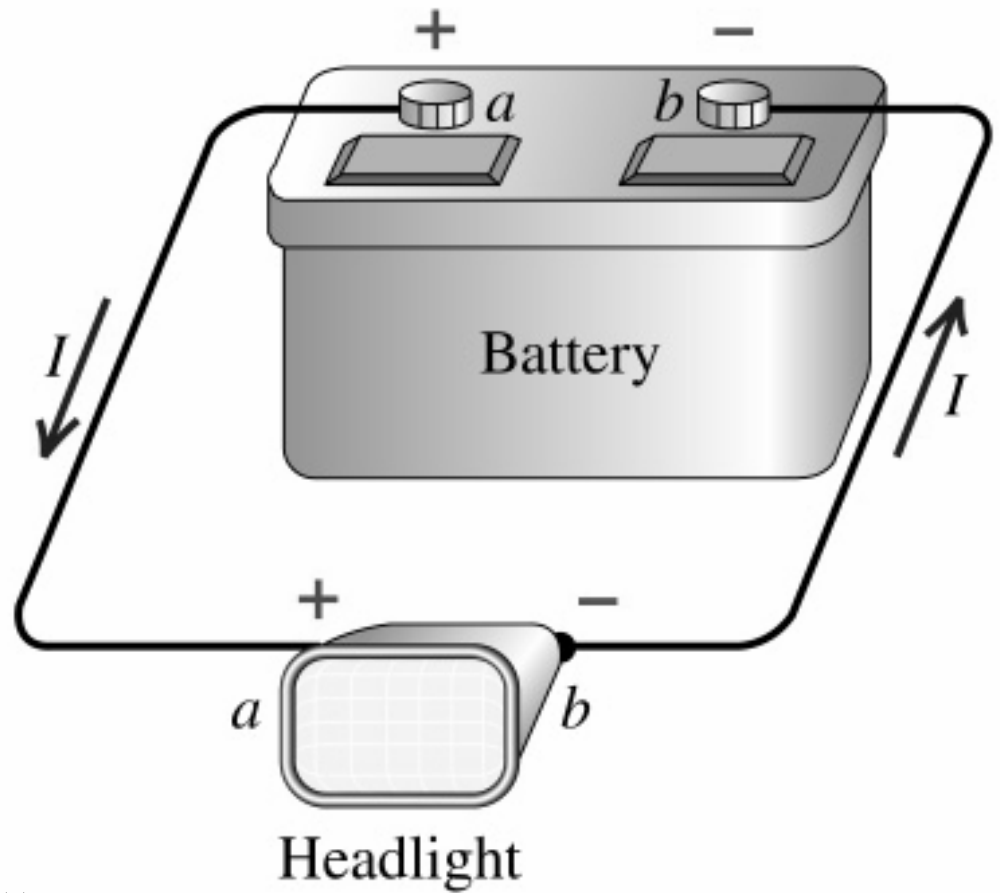
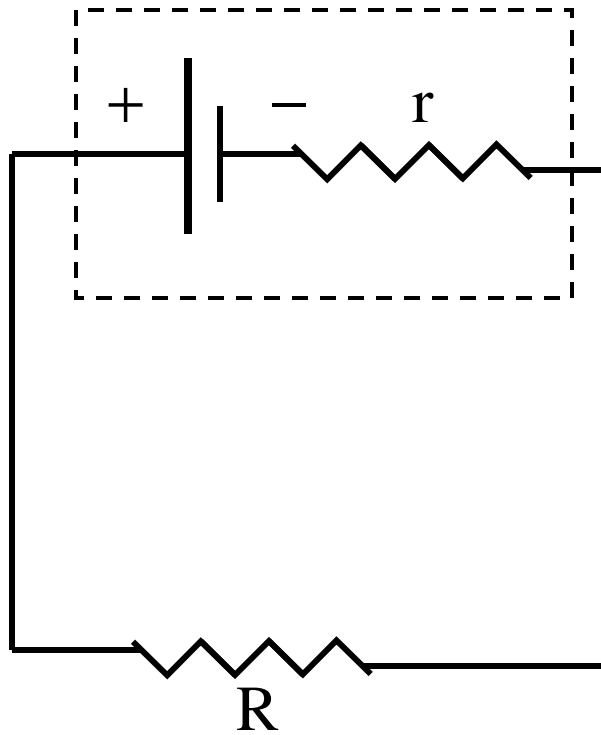


In series



Look back at the last example

Real life example of previous circuit is a car battery and a headlamp



$R$  = resistance in headlight  
 $r$  = internal resistance in battery



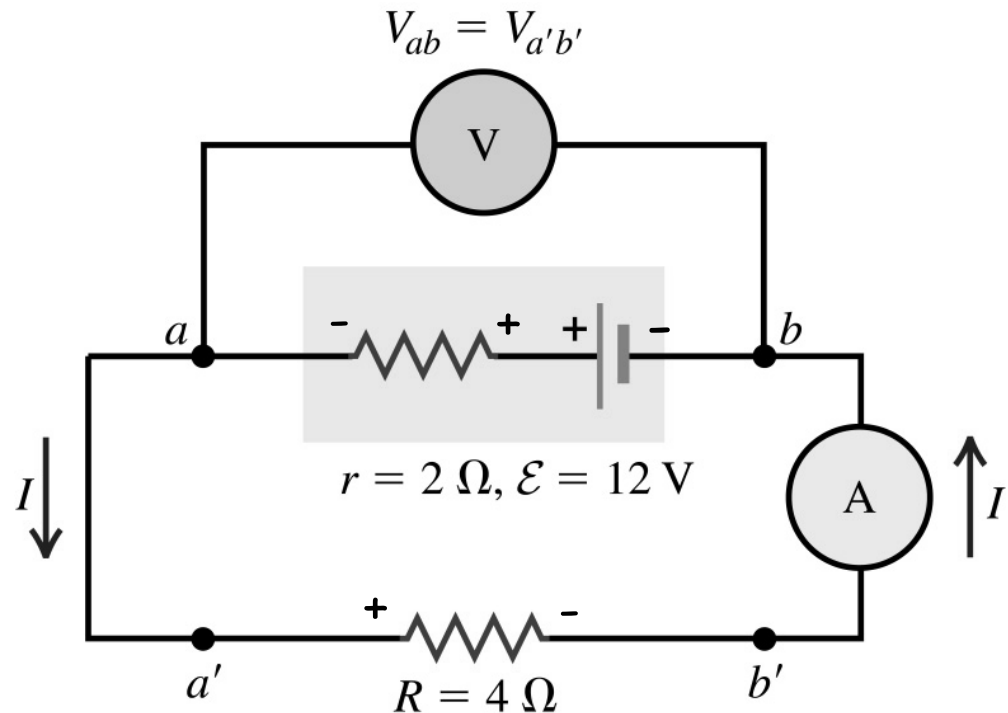
Real Battery with 12 Volts and an internal resistance of 2 ohms in a circuit with a resistor of 4 ohms

$$\mathcal{E} - Ir = IR$$

Rearranging:

$$\mathcal{E} - Ir - IR = 0$$

Can interpret this as the sum of all potential differences around a closed loop must add to zero.



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Kirchoff's voltage law or loop rule.

Can define a voltage rise as positive OR a voltage fall as positive.

Real Battery with 12 volts and an internal resistance of  $2\Omega$  in a circuit with a resistor of  $4\Omega$

$$V_{AB} = V_{A'B'}$$

$$\mathcal{E} - Ir = IR$$

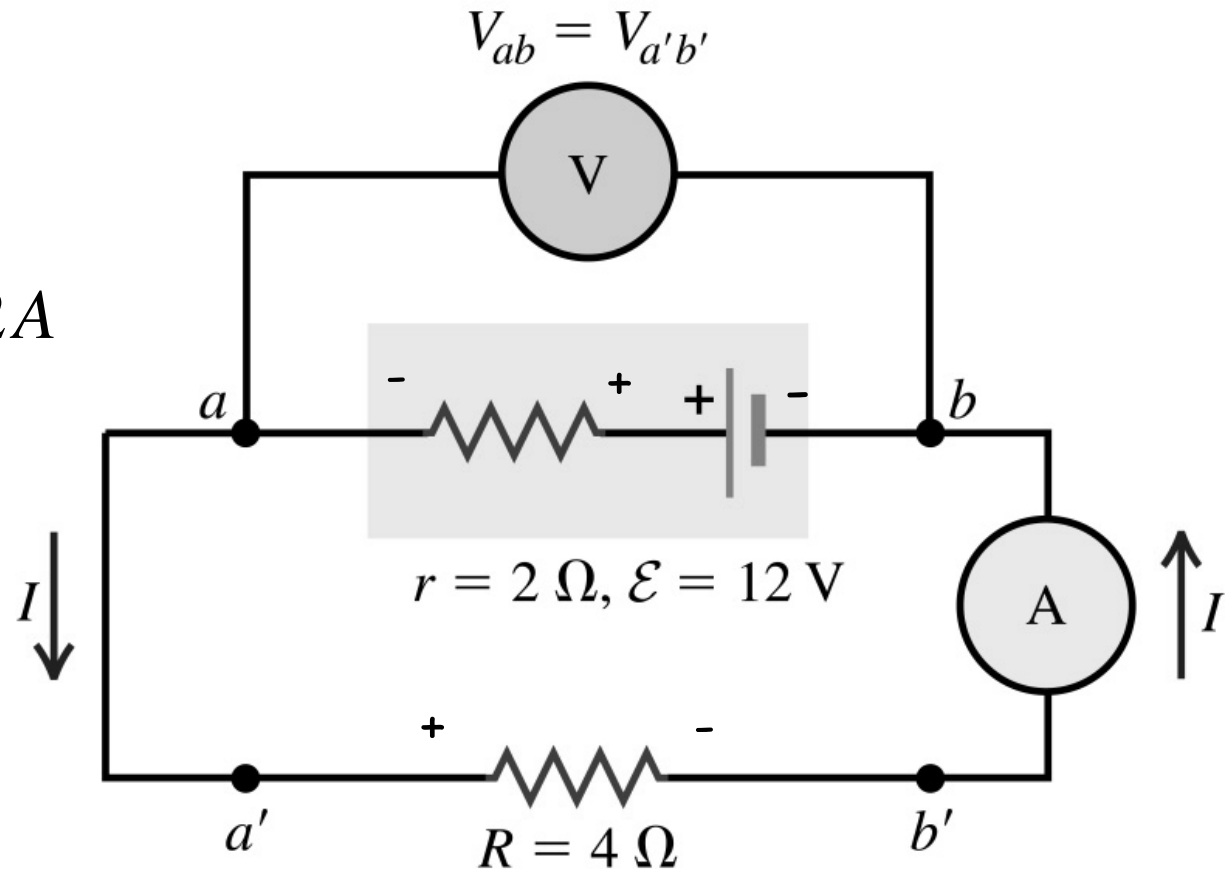
$$I(R + r) = \mathcal{E}$$

$$I = \frac{\mathcal{E}}{R + r} = \frac{12V}{6\Omega} = 2A$$

Voltage drop across  $2\Omega$  is 4 volts

Voltage drop across  $4\Omega$  is 8 volts

$$V_{ab} = V_{a'b'} = 8 \text{ volts}$$



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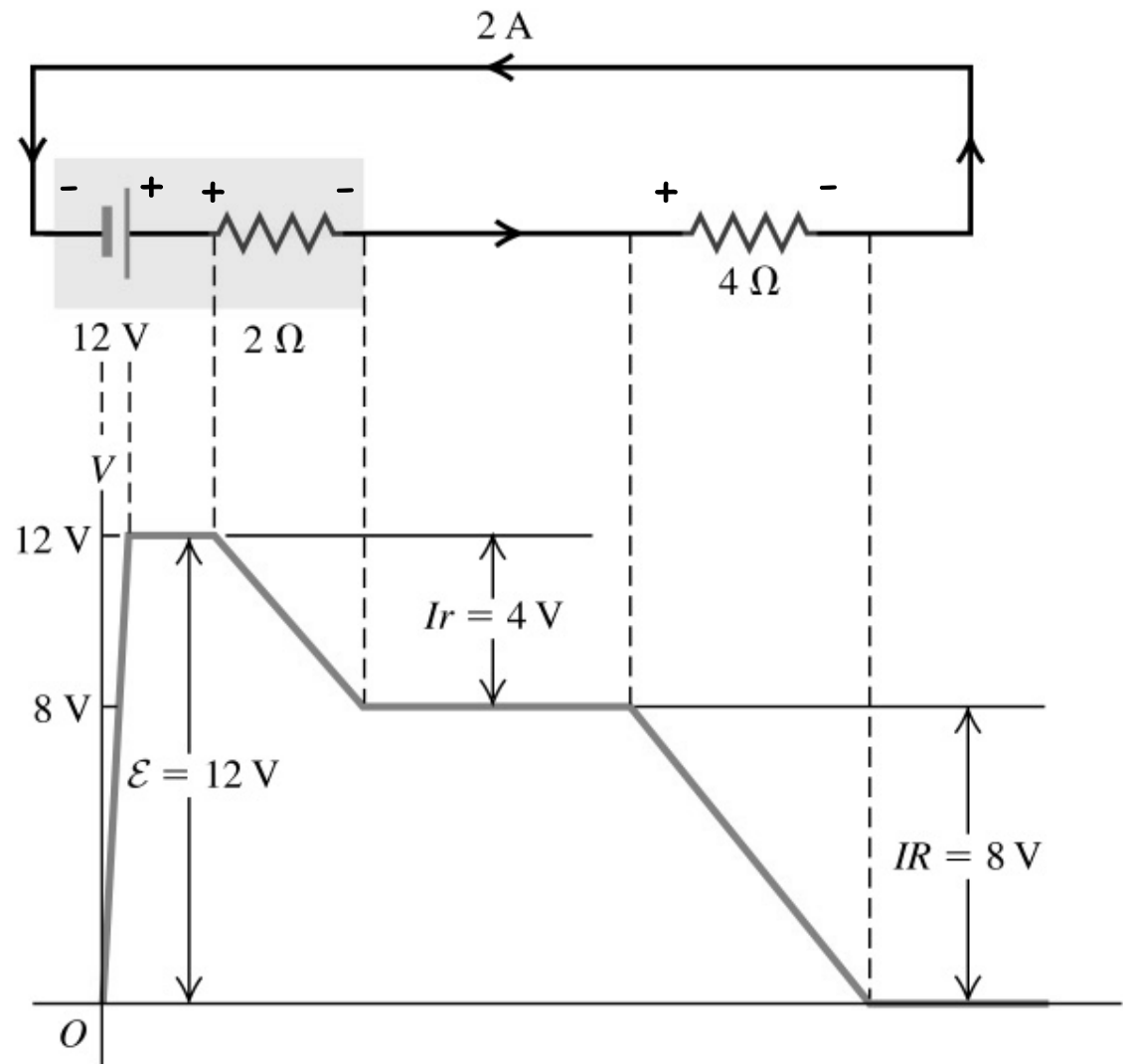
Terminal potential difference  $V_{AB} = 12 - 4 = 8\text{ V}$

# Diagram of Electric Potential/Voltage in the circuit

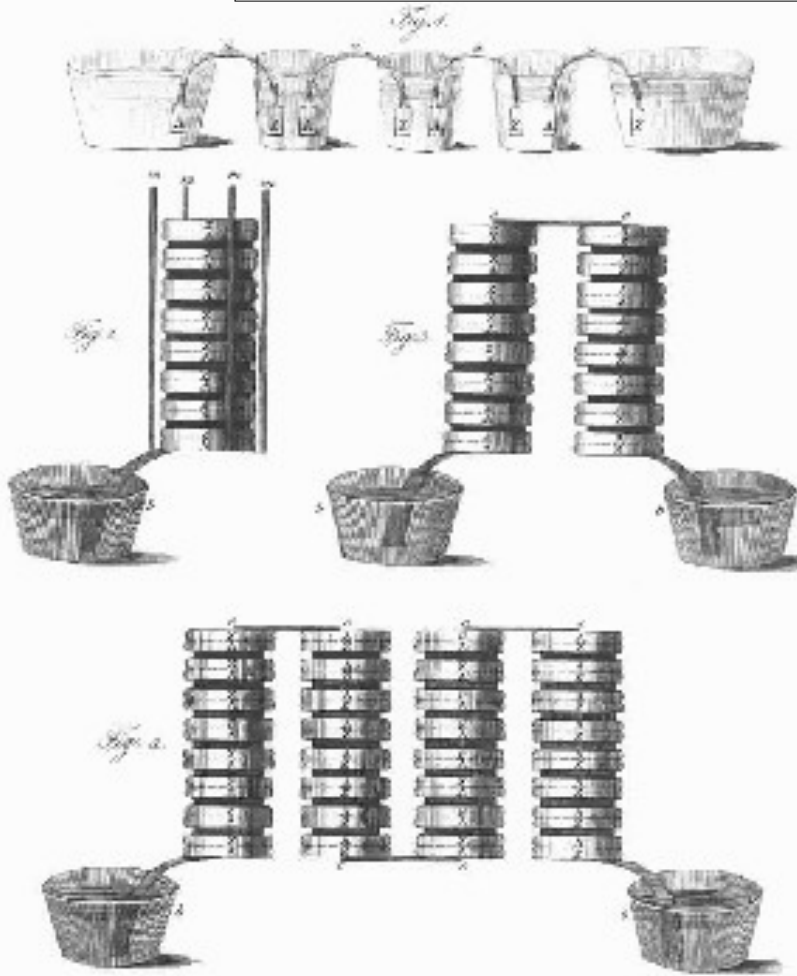
Voltage drop across 2 Ohms is 4 volts

Voltage drop across 4 Ohms is 8 volts

NOTE  $4\Omega$  resistor has only 8 of the 12 volts.



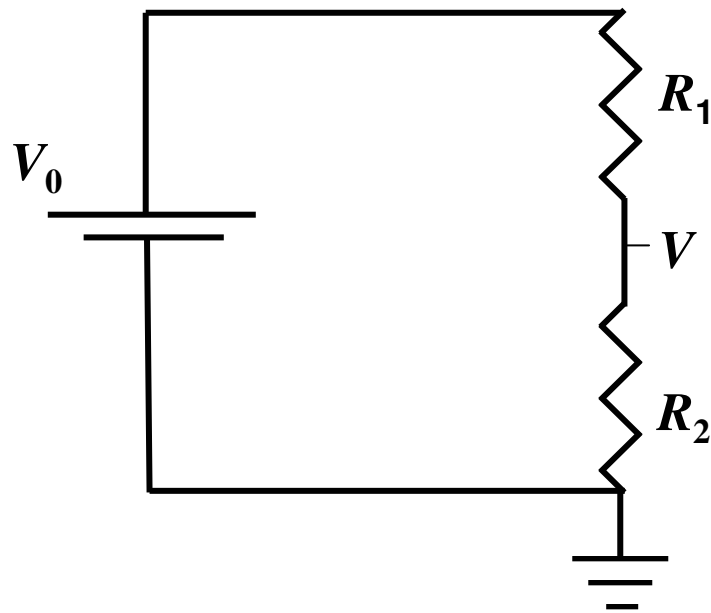
## Drawing of Volta's Pile or Battery



Demonstrating before Napoleon

Manuscript on the invention of the battery sent to the Royal Society of London. Cardboard soaked in salt water was placed in between the zinc and silver disks.

# The World's Simplest (and most useful) circuit: Voltage Divider



$$V = ?$$

$$V = IR_2 = \left( \frac{V_0}{R_1 + R_2} \right) R_2$$

$$R_2 \ll R_1 \quad V=0$$

$$R_2 = R_1 \quad V = \frac{V_0}{2}$$

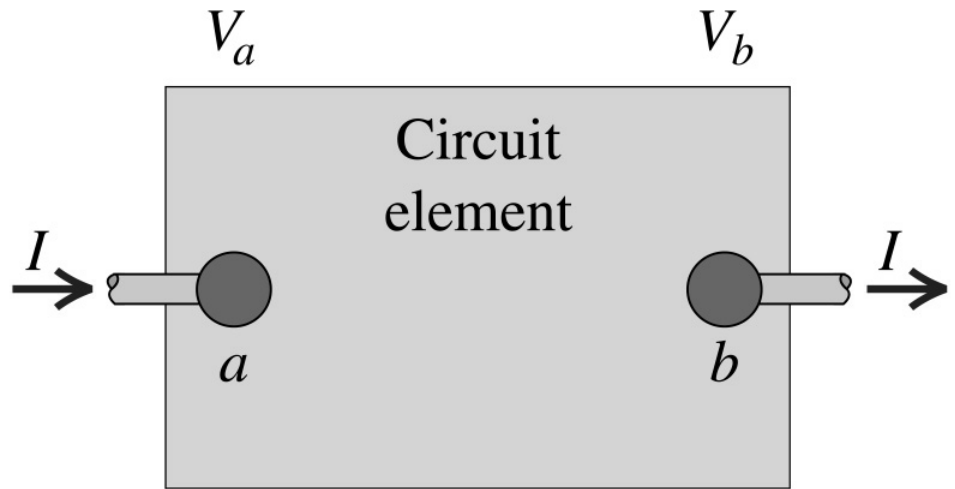
$$R_2 \gg R_1 \quad V = V_0$$

**By varying  $R_2$  we can controllably adjust the output voltage!**

# Power

Very important

Suppose we have a circuit element that has a voltage drop of  $V_{ab} = V_a - V_b$  and a current flow of  $I$ .



What is the change in potential energy in this circuit element?

$$dU = (V_{ab}) dq$$

What is the time rate change in potential energy in this circuit element?

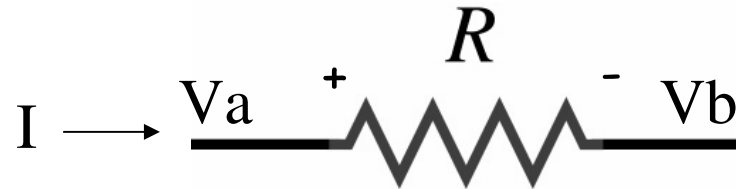
$$P = \frac{dU}{dt} = (V_{ab}) \frac{dq}{dt} = (V_{ab}) I$$

Power,  $P$ , is the time rate change in energy and equals voltage  $\times$  current

$$P = (V_{ab}) I \quad \text{Units = voltage-amps = Watts}$$

Power

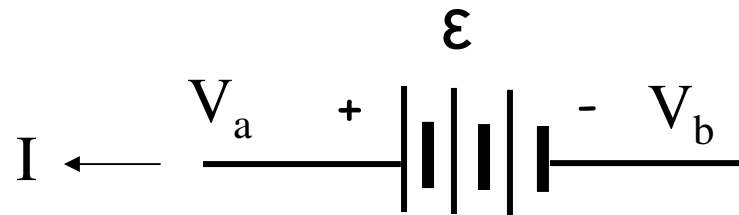
Resistor



$$\text{Power } P = I (V_a - V_b) = IV = I (IR) = I^2R = V^2/R$$

Electrical power converted to Joule heat.

EMF



$$\text{Power } P = I (V_a - V_b) = I \epsilon$$

Electrical power can be + or - depending on direction of  $I$ .

## Power Example of Battery and resistor

What is the power in resistor R?

$$I^2 R = 2 * 2 * 4 = 16 \text{ W}$$

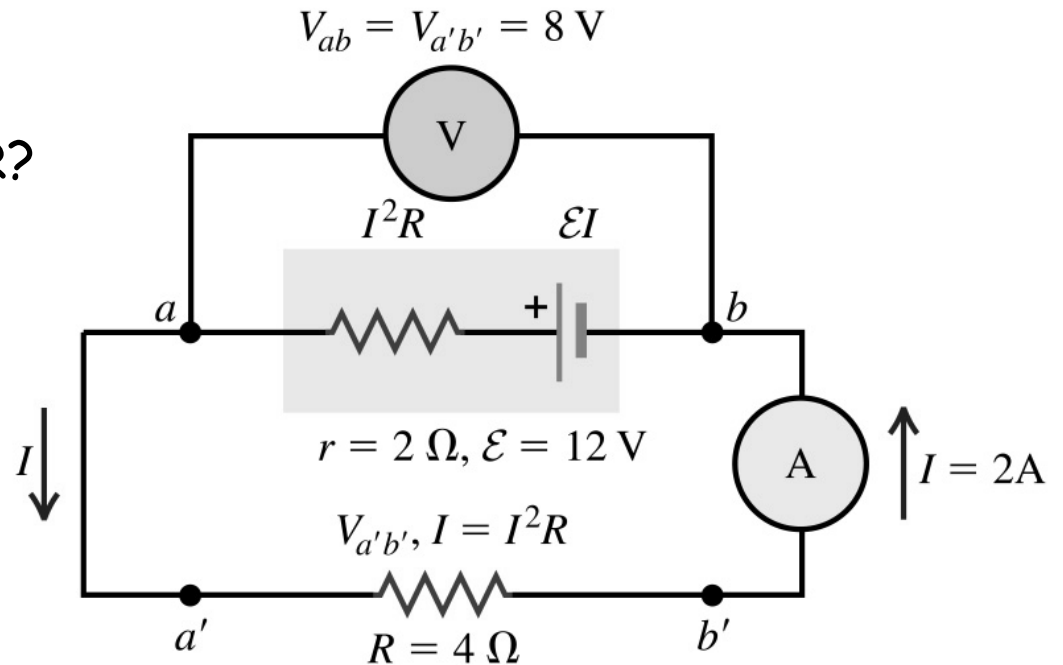
What is the power in internal resistance r?

$$I^2 r = 2 * 2 * 2 = 8 \text{ W}$$

What is the rate of energy conversion of the battery?

$$\mathcal{E} I = 12 * 2 = 24 \text{ W}$$

=> Energy conversion rate equals sum of power in both R and r



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## Y&F Problem 25.49

The capacity of a storage battery in your car is rated in amp-hours. A 12 volt battery rated at  $50 \cdot A \cdot h$ , can supply 50 amps for 1 hour at 12 volts or 25 amps for 2 hours etc.

A.) What is the total energy supplied by this battery?

$$P = \frac{dU}{dt} = IV = (50A)(12V) = 600W$$

$$U = Pt = (600W)(1hr)(3600s/hr) = 2.16MJ$$

B.) if a electric battery charger supplies 0.45 kW, how long does it take to fully charge a dead battery?

$$U = Pt$$

$$t = U / P = (2.16MJ) / (0.45kW)$$

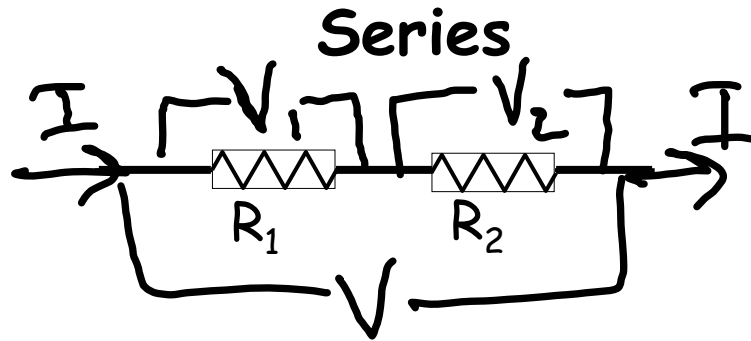
$$= 4680s = 78m$$

## Georg Simon Ohm (1789-1854)



Born in Bavaria. At the university in 1805, he spent too much time dancing, ice skating and playing billiards and left after 3 semesters. Eventually he returned and graduated in 1811. He taught in various high schools and studied privately. He published *Die galvanische Kette, mathematisch bearbeitet* (1827) where his famous law was described. He had difficulty obtaining a university position, but eventually he was recognized by the Royal Society and other academies. In the last 2 years of his life, he was professor at the University of Munich.

# How to combine resistors



**Wiring**

Each resistor on the same wire.

**Voltage**

Different for each resistor.

$$V_{\text{total}} = V_1 + V_2$$

**Current**

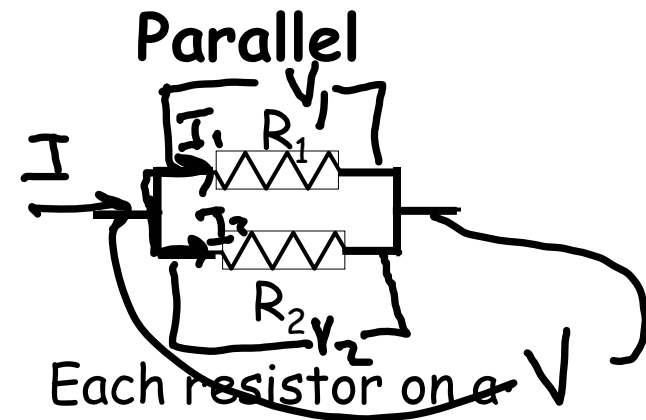
Same for each resistor

$$I_{\text{total}} = I_1 = I_2$$

**Resistance**

Increases

$$R_{\text{eq}} = R_1 + R_2$$



Each resistor on a different wire.

Same for each resistor.

$$V_{\text{total}} = V_1 = V_2$$

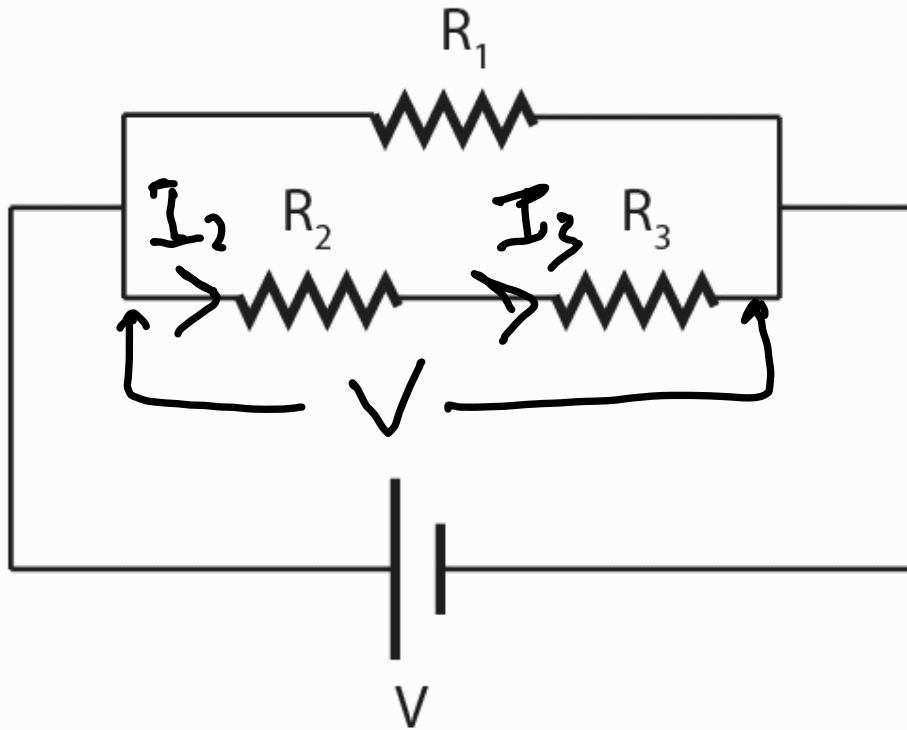
Different for each resistor

$$I_{\text{total}} = I_1 + I_2$$

Decreases

$$1/R_{\text{eq}} = 1/R_1 + 1/R_2$$

6) Three resistors are connected to a battery with emf  $V$  as shown. The resistances of the resistors are all the same, i.e.  $R_1 = R_2 = R_3 = R$ .



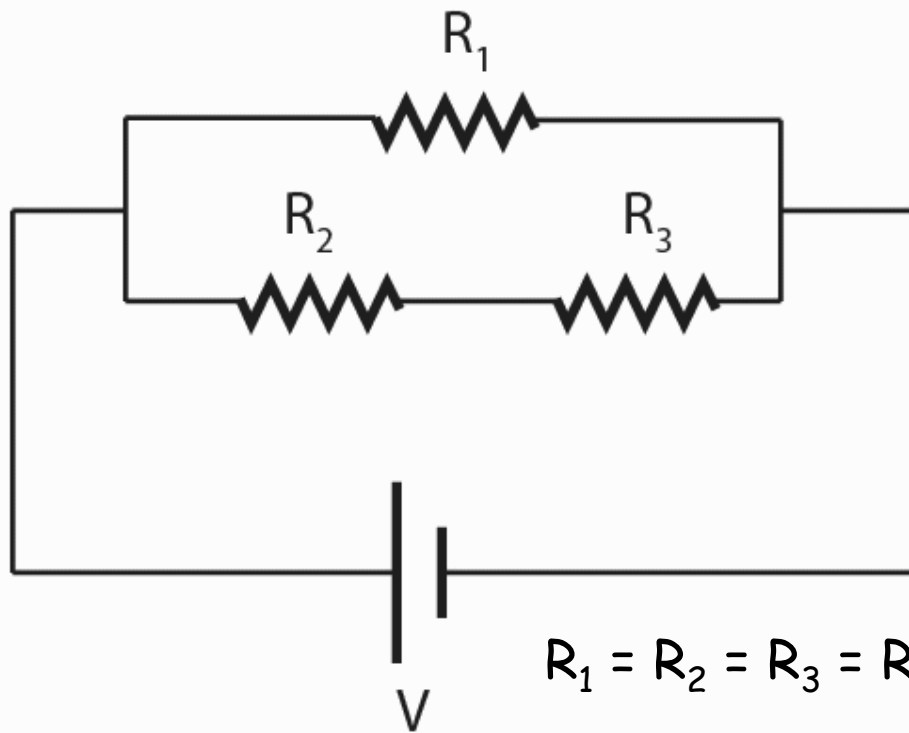
# Clicker problem

Compare the current through  $R_2$  with the current through  $R_3$

- a)  $I_2 > I_3$     **b)  $I_2 = I_3$**     c)  $I_2 < I_3$

$R_2$  in series with  $R_3$   $\Rightarrow$  Current through  $R_2$  and  $R_3$  is the same

$$I = V / (R_1 + R_2)$$



Analyze this

Compare the current through  $R_1$  with the current through  $R_2$

$$I_1/I_2 = 1/2$$

$$I_1/I_2 = 1/3$$

$$I_1/I_2 = 1$$

$$I_1/I_2 = 2$$

$$I_1/I_2 = 3$$

Compare the voltage across  $R_2$  with the voltage across  $R_3$

$$V_2 > V_3$$

$$V_2 = V_3 = V$$

$$V_2 = V_3 < V$$

$$V_2 < V_3$$

Compare the voltage across  $R_1$  with the voltage across  $R_2$

$$V_1 = V_2 = V$$

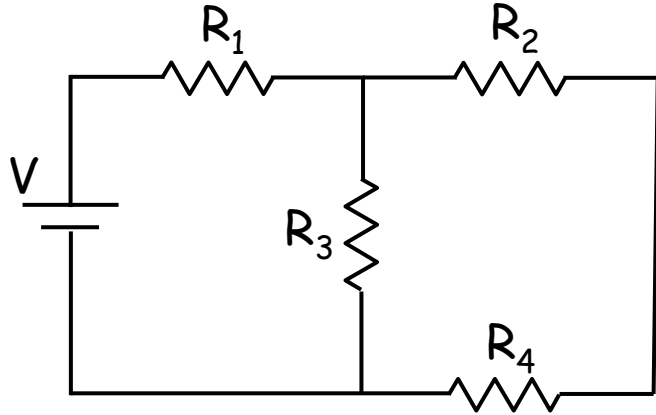
$$V_1 = \frac{1}{2} V_2 = V$$

$$V_1 = 2V_2 = V$$

$$V_1 = \frac{1}{2} V_2 = 1/5 V$$

$$V_1 = \frac{1}{2} V_2 = \frac{1}{2} V$$

# How do we handle complex problems ?



In the circuit shown:  $V = 18\text{V}$ ,  
 $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $R_3 = 3\Omega$ , and  
 $R_4 = 4\Omega$ .

What is  $V_2$ , the voltage  
across  $R_2$ ?

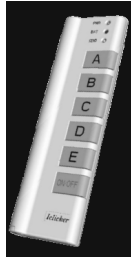
- Conceptual Analysis:

- Ohm's Law: when current  $I$  flows through resistance  $R$ , the potential drop  $V$  is given by:  $V = IR$ .
- Resistances are combined in series and parallel combinations
  - $R_{\text{series}} = R_a + R_b$
  - $(1/R_{\text{parallel}}) = (1/R_a) + (1/R_b)$

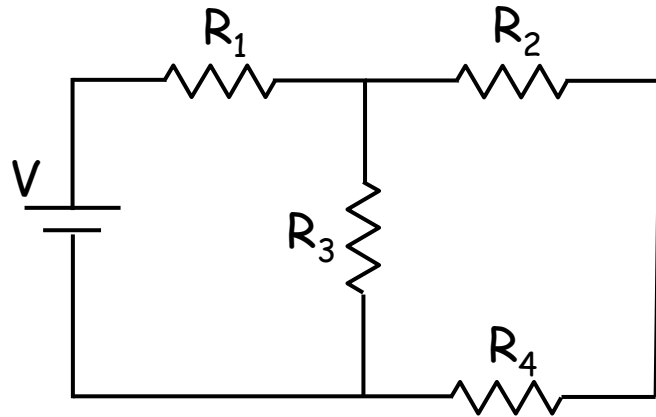
Divide and Conquer



# Clicker example



BB



In the circuit shown:  $V = 18\text{V}$ ,  
 $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $R_3 = 3\Omega$ , and  
 $R_4 = 4\Omega$ .

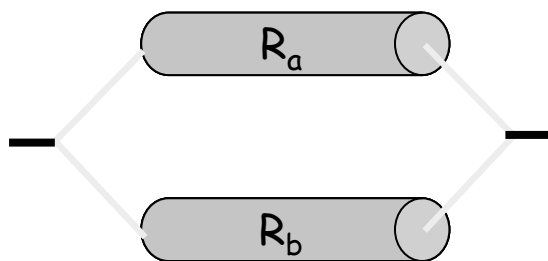
- **Combine Resistances:**

$R_1$  and  $R_2$  are connected:

(A) in series (B) in parallel

(C) neither in series nor in parallel

Parallel Combination



Parallel: Can make a loop that contains only those two resistors

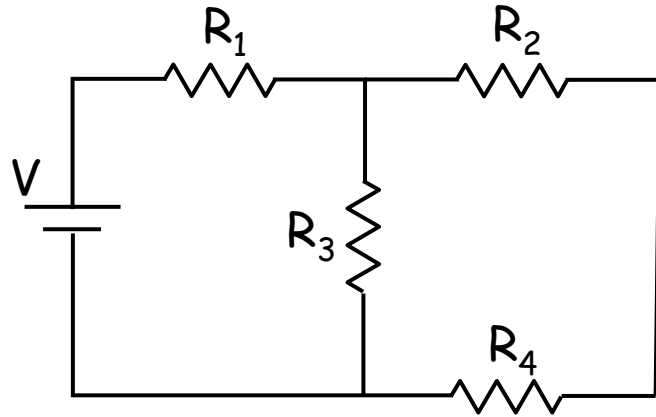
Series Combination



Series : Every loop with resistor 1 also has resistor 2.



# Clicker Problem



In the circuit shown:  $V = 18\text{V}$ ,  
 $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $R_3 = 3\Omega$ , and  
 $R_4 = 4\Omega$ .



- Combine Resistances:

$R_2$  and  $R_4$  are connected:

(A) in series (B) in parallel (C) neither in series nor in parallel

What is  $V_2$ , the voltage across  $R_2$ ?

## Series Combination

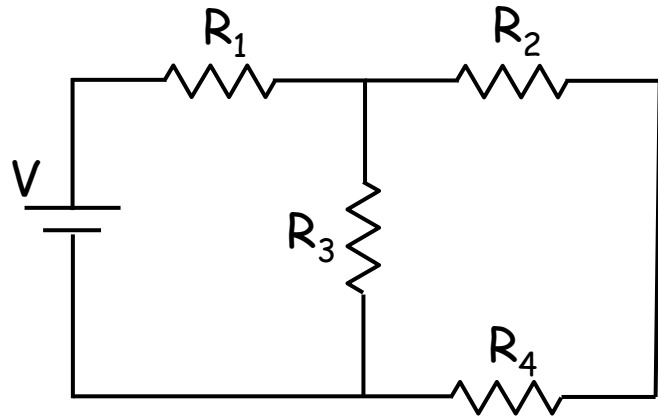


Series : Every loop with resistor 1 also has resistor 2.





# Clicker problem



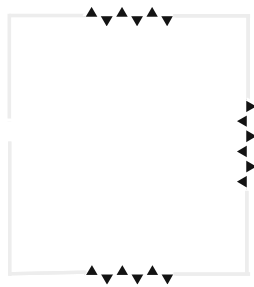
In the circuit shown:  $V = 18\text{V}$ ,  
 $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $R_3 = 3\Omega$ , and  
 $R_4 = 4\Omega$ .

What is  $V_2$ , the voltage  
across  $R_2$ ?

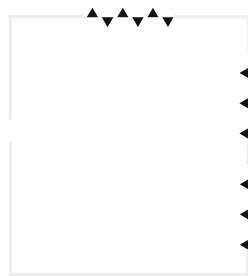
- Combine Resistances:

Redraw the circuit using the equivalent resistor  $R_{24}$  = series combination of  $R_2$  and  $R_4$ .

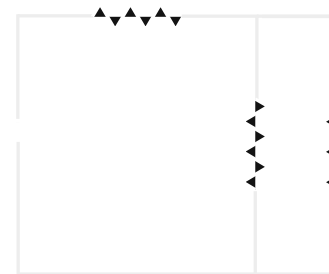
$R_2$  and  $R_4$  are connected in series =  $R_{24} = 2 + 4 = 6\Omega$



(A)



(B)



(C)



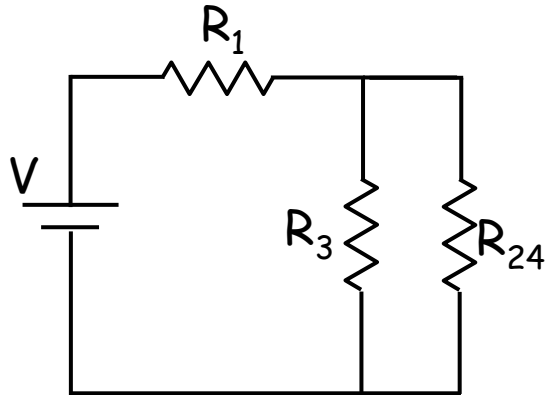
BB



# Clicker example



BB



In the circuit shown:  $V = 18\text{V}$ ,  
 $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ ,  $R_3 = 3\Omega$ , and  
 $R_4 = 4\Omega$ .

$$R_{24} = 6\Omega$$

What is  $V_2$ , the voltage  
across  $R_2$ ?

• Combine Resistances:

What is the value of  $R_{234}$ ?

(A)  $R_{234} = 1\Omega$     (B)  $R_{234} = 2\Omega$     (C)  $R_{234} = 4\Omega$     (D)  $R_{234} = 6\Omega$

$R_2$  and  $R_4$  are connected in series =  $R_{24}$   
 $R_3$  and  $R_{24}$  are connected in parallel =  $R_{234}$

$$(1/R_{\text{parallel}}) = (1/R_a) + (1/R_b) \implies 1/R_{234} = (1/3) + (1/6) = (3/6)\Omega^{-1}$$

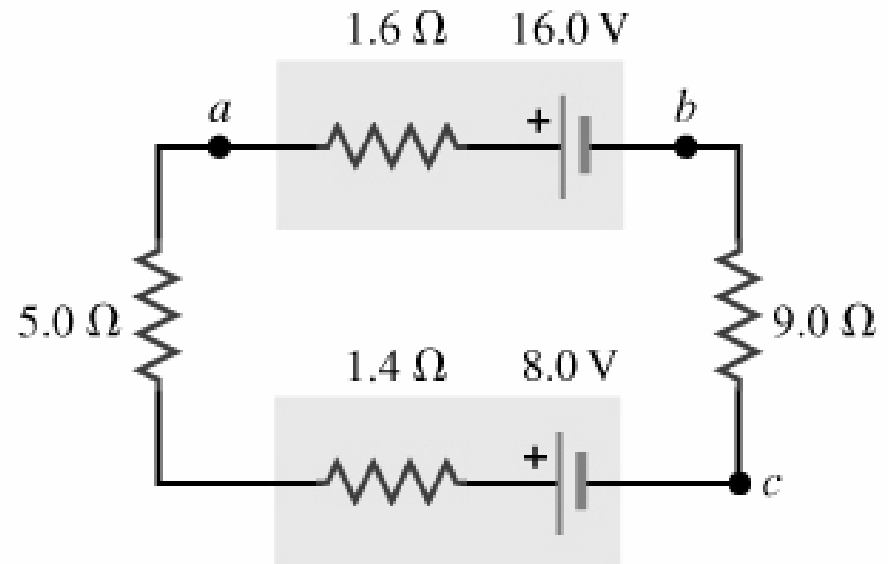
$$\implies R_{234} = 2\Omega$$



## Y&F 25.36

The circuit shown in the figure contains two batteries, each with an emf and an internal resistance, and two resistors.

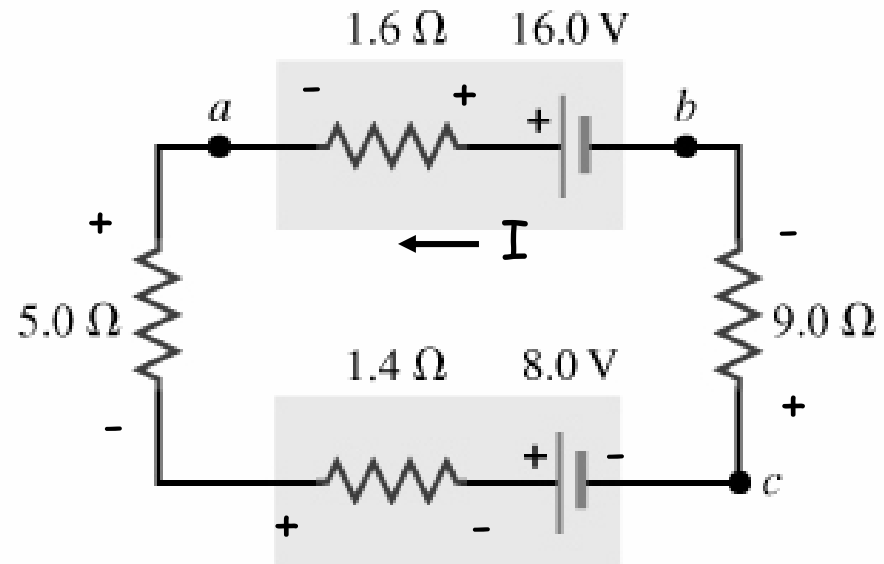
- Find the direction and magnitude of the current in the circuit
- Find the terminal voltage  $V_{ab}$  of the 16.0-V battery.
- Find the potential difference  $V_{bc}$  of point b with respect to point c .



## Y&F 25.36 (not a clicker problem)

The circuit shown in the figure contains two batteries, each with an emf and an internal resistance, and two resistors.

- Find the direction and magnitude of the current in the circuit
- Find the terminal voltage  $V_{ab}$  of the 16.0-V battery.
- Find the potential difference  $V_{bc}$  of point b with respect to point c.



A.) Use Kirchoff's loop law:

$$16V - 1.6\Omega I - 5.0\Omega I - 1.4\Omega I - 8V - 9.0\Omega I = 0$$

$$8V - 17\Omega I = 0$$

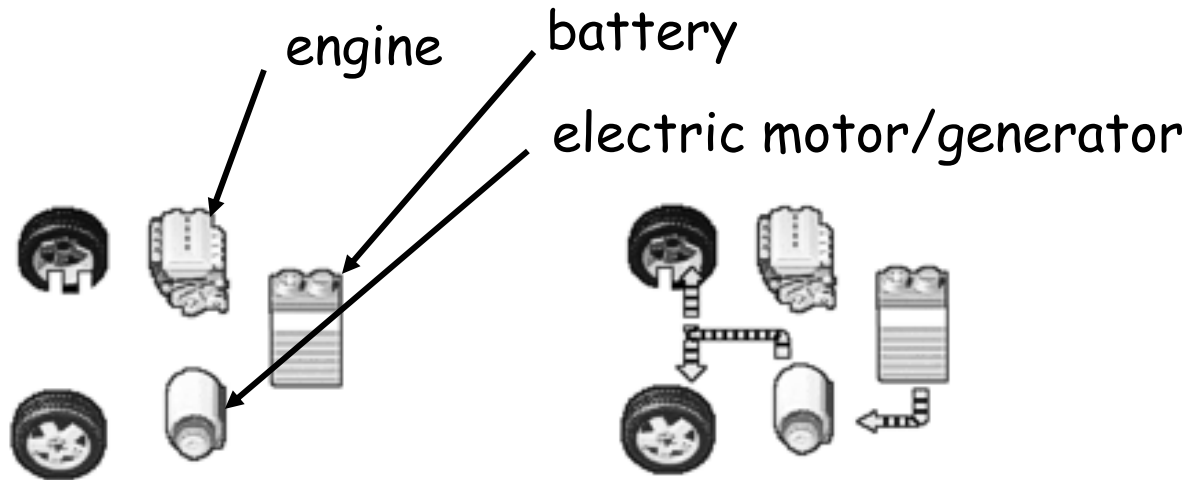
$$I = \frac{8}{17} A = 0.471 A$$

B.)  $V_{ab} = 16V - I(1.6\Omega) = 16V - (0.471 A)(1.6\Omega) = 15.2 V$

C.)  $V_{bc} = -I(9.0\Omega) = -(0.471 A)(9.0\Omega) = -4.24 V$

# Prius - hybrid car and "energy machine".

display



## Standby

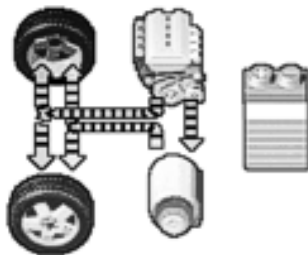
While waiting at a stoplight, the engine will shut off. The resulting silence and lack of vibration is a pleasure that adds to the driving experience.

## Battery Only

Propulsion is sometimes provided exclusively using the electric motor & battery-pack. The engine can shut off during this time.

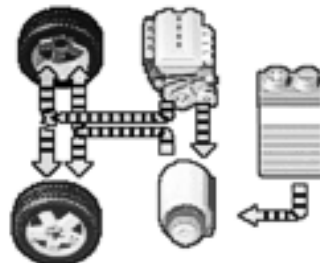
## Highway Cruising

This is the most common behavior while traveling at a constant speed on the highway. This electrical activity often surprises those researching how Prius operates.



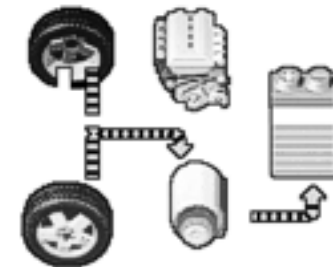
## Moderate Acceleration

This configuration is very common when accelerating, including when you need to merge onto a highway. This is also the method Prius uses to climb most hills.



## Full Power & Slowing

This mode occurs in 2 very different situations, when maximum thrust is needed and when gradual slowing occurs.



## Regenerating

When you step on the brake or remove your foot from the accelerator, the system will automatically convert the kinetic energy to electricity for the battery-pack.

# Prius - hybrid car

## Battery Pack

- 99 lbs
- NiMH
- 168 cells
- 1.2 V each cell
- 6.5 Ah total
- 201.6 V total

## Electric Motor

- 50 kW power
- 67 hp at 1200 - 1540 rpm
- 500 V maximum

## Rated MPG

- 60 city
- 51 highway



## Gas Engine

- 1.5 L
- 4 cylinder - 16 valve
- 76 HP at 5000 RPM

## Brakes

- regenerative (primary)
- hydraulic (secondary)
- ABS (anti lock)

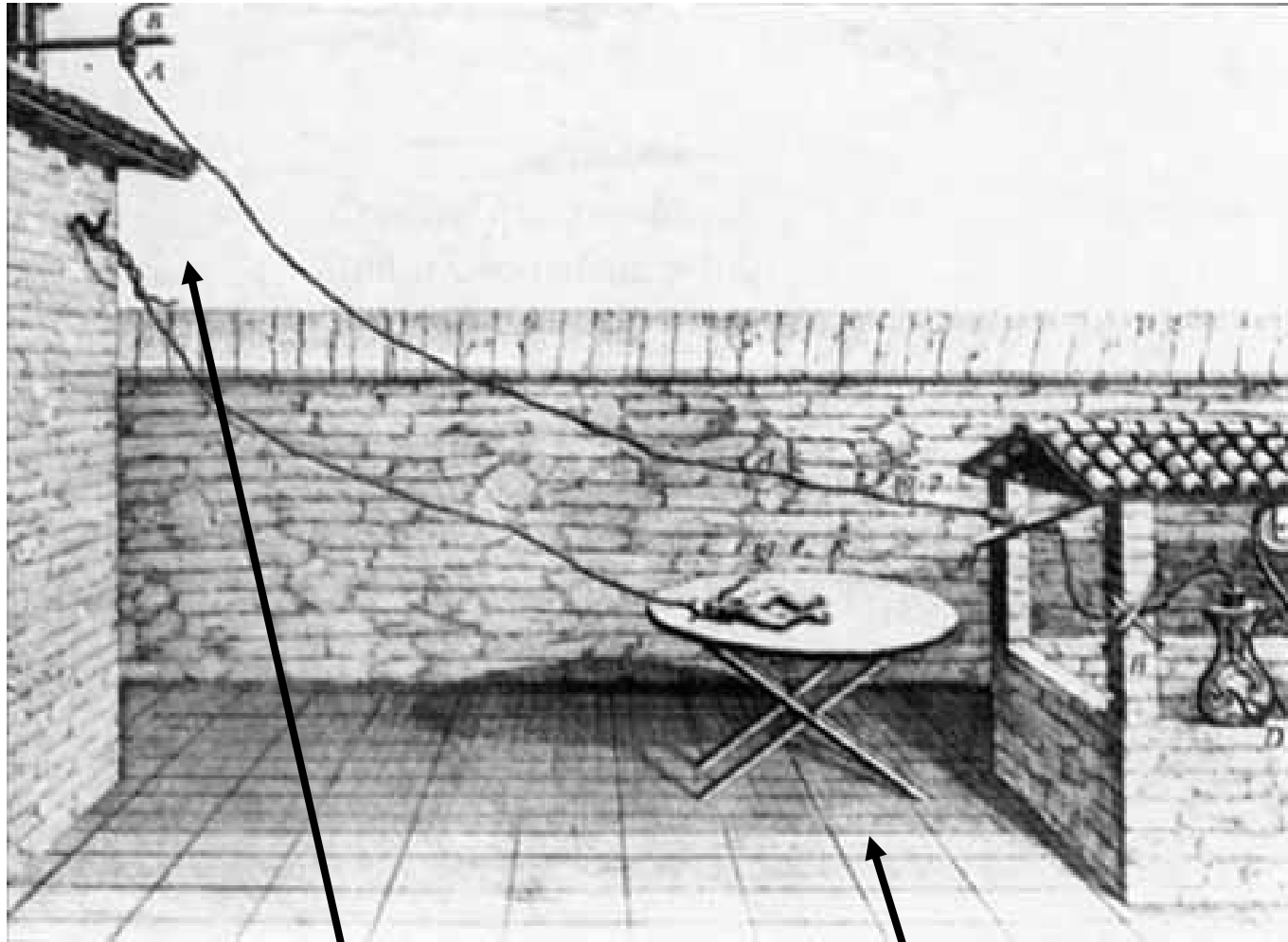
## Early History of Current and Batteries

The development of the battery has a curious history. Henry Cavendish studied sting rays (called Torpedoes) to investigate how they created electricity around 1776. Around ~1790, Luigi Galvani applied current to frog's legs and observed them twitching. He believed there was an "animal electricity" stored in the frog's brain. About 1800, Alessandro Volta, invented the "pile", a series of silver and zinc disks in salt water. Volta tested his electricity by attaching electrodes to his tongue and eventually demonstrated his pile before Napoleon.

In 1803, George Forster, a convicted murderer was hanged at Newgate prison, and Galvani's nephew, Giovanni Aldini, applied current from a Volta pile to the corpse causing the jaws, eyes, hands and legs to move. This process of resuscitation was eventually applied to drowning victims. This created philosophical and religious controversy about life, death and electricity. This gave Mary Shelley the idea for the 1818 novel, Frankenstein, about a corpse brought to life by electricity.



## Drawing of one of Luigi Galvani's experiments



Leyden  
Jar

lightning rod attached to froggy!!