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VTU

Common to All Branches

1 & 2 Semester

Physics Cycle

2010 Scheme

Basic Electrical Engineering

[10ELE15]

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Branch Name: Common to all branches
SEM: 1/2
University: VTU
Syllabus: 2010

Table of Contents:

Basic Electrical Engineering (10ELE15):

BASIC ELECTRICAL ENGINEERING

PART – A

Unit-I

1–a) D. C. Circuits: Ohm's Law and Kirchhoff's Laws, analysis of series, parallel and series- parallel circuits excited by independent voltage sources. Power and Energy. Illustrative examples. **04Hours**

I–b) Electromagnetism: Faradays Laws, Lenz's Law, Fleming's Rules, Statically and dynamically induced emf's. Concept of self inductance, mutual inductance and coefficient of coupling. Energy stored in magnetic field. Illustrative examples. **03Hours**

Unit-II

2. Single-phase A.C. Circuits: Generation of sinusoidal voltage, definition of average value, root mean square value, form factor and peak factor of sinusoidally varying voltage and current, phasor representation of alternating quantities. Analysis, with phasor diagrams, of R, L, C, R-L, R-C and R-L-C circuits, real power, reactive power, apparent power and power factor. Illustrative examples involving series, parallel and series-parallel circuits **07 Hours**

Unit-III

3 Three Phase Circuits: Necessity and advantages of three phase systems, generation of three phase power, definition of Phase sequence, balanced supply and balanced load. Relationship between line and phase values of balanced star and delta connections. Power in balanced three-phase circuits, measurement of power by two-wattmeter method. Illustrative examples. **06 Hours**

Unit-IV

4–a) Measuring Instruments: Construction and Principle of operation of dynamometer type wattmeter and single-phase induction type energy meter (problems excluded). **03 Hours**

4–b) Domestic Wiring: Service mains, meter board and distribution board. Brief discussion on Cleat, Casing & Capping and conduit (concealed) wiring. Two-way and three-way control of a lamp. Elementary discussion on fuse and Miniature Circuit Breaker (MCB's). Electric shock, precautions against shock –Earthing: Pipe and Plate.
03 Hours

PART – B

Unit-V

5. DC Machines: Working principle of DC machine as a generator and a motor. Types and constructional features. emf equation of generator, relation between emf induced and terminal voltage enumerating the brush drop and drop due to armature reaction. Illustrative examples.
DC motor working principle, Back emf and its significance, torque equation. Types of D.C. motors, characteristics and applications. Necessity of a starter for DC motor. Illustrative examples on back emf and torque. **07 Hours**

Unit-VI
6. Transformers: Principle of operation and construction of single-phase transformers (core and shell types). emf equation, losses, efficiency and voltage regulation (Open Circuit and Short circuit tests, equivalent circuit and phasor diagrams are excluded). Illustrative problems on emf equation and efficiency only. **07 H**ours

Unit-VII

7. Synchronous Generators: Principle of operation. Types and constructional features. emf equation. Concept of winding factor (excluding derivation of distribution and pitch factors). Illustrative examples on emf. equation. **06 Hours**

Unit-VIII

8. Three Phase Induction Motors: Concept of rotating magnetic field. Principle of operation. Types and Constructional features. Slip and its significance. Applications of squirrel - cage and slip - ring motors. Necessity of a starter, star-delta starter. Illustrative examples on slip calculations. **06 Hours**

TEXT BOOKS

1 "Basic Electrical Engineering", D C Kulshreshtha, TMH, 2009 Edition.

2 "Fundamentals of Electrical Engineering", Rajendra Prasad, PHI, Second Edition, 2009.

REFERENCE BOOKS:

1. "Electrical Technology", E. Hughes International Students 9th Edition, Pearson, 2005.

2. "Basic Electrical Engineering", Abhijit Chakrabarti, Sudiptanath, Chandan Kumar Chanda, TMH, First reprint 2009.

3. Problems in Electrical Engineering, Parker Smith, CBS Publishers and Distributors, 9th Edition, 2003.

Contents

PART – A

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D. C. Circuits

Ohm's law and state its limitations.

Ohm's Law : the current flowing through the electric the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provided the temperature remains constant.

Limitations of Ohm's Law

The limitations of the Ohm's law are,

1) It is not applicable to the nonlinear devices such as diodes, zener diodes, voltage regulators ect.
2) It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is given by,

$$V = K I^m \quad \text{where k, m are constants.}$$

(I) Current is what flows on a wire or conductor like water flowing down a river. Current flows from negative to positive on the surface of a conductor. Current is measured in (A) amperes or amps.
(E) Voltage Ohm's Law defines the relationships between (P) power, (E) voltage, (I) current, and (R) resistance. One ohm is the resistance value through which one volt will maintain a current of one ampere is the difference in electrical potential between two points in a circuit. It's the push or pressure behind current flow through a circuit, and is measured in (V) volts.

(R) Resistance determines how much current will flow through a component. Resistors are used to control voltage and current levels. A very high resistance allows a small amount of current to flow. A very low resistance allows a large amount of current to flow. Resistance is measured in Ω ohms.
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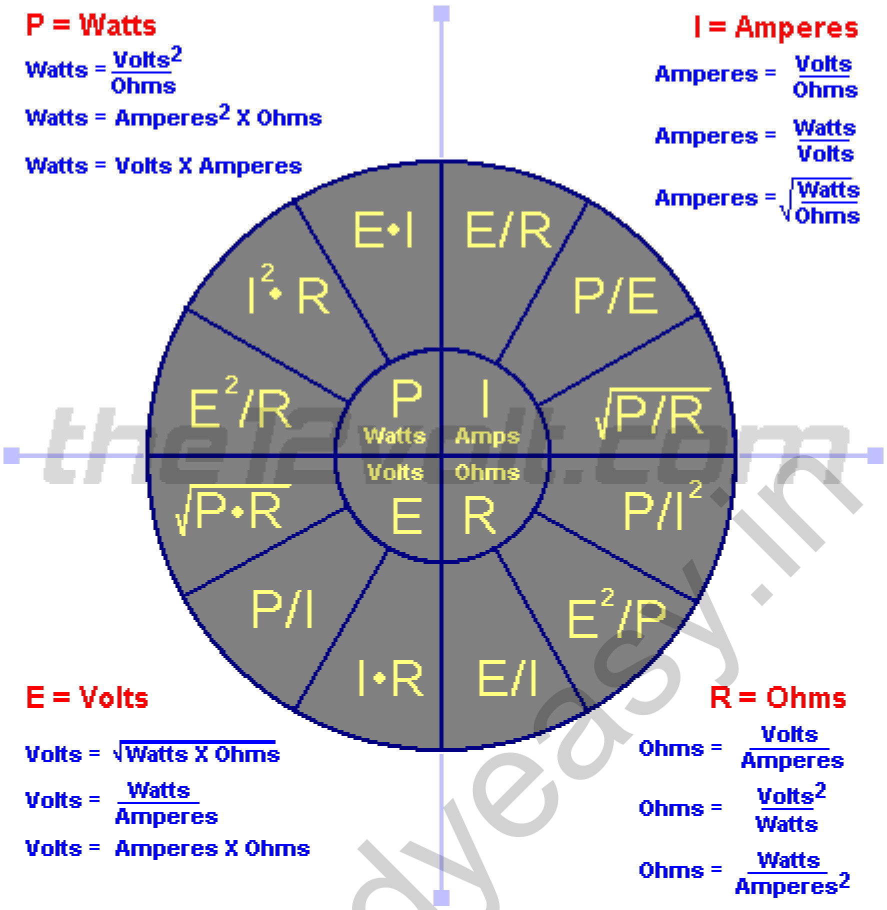

To make a current flow through a resistance there must be a voltage across that resistance. Ohm's Law shows the relationship between the voltage (V), current (I) and resistance (R). It can be written in three ways:

$\mathbf{V = I \times R}$ or $\mathbf{I = \frac{V}{R}}$ or $\mathbf{R = \frac{V}{I}}$

where: V = voltage in volts (V)
I = current in amps (A)
R = resistance in ohms (Ω)

or: V = voltage in volts (V)
I = current in milliamps (mA)
R = resistance in kilohms (kΩ)

The VIR triangle

V

I R

Ohm's Law

State and explain Kirchhoff's laws.

Kirchhoff's current law

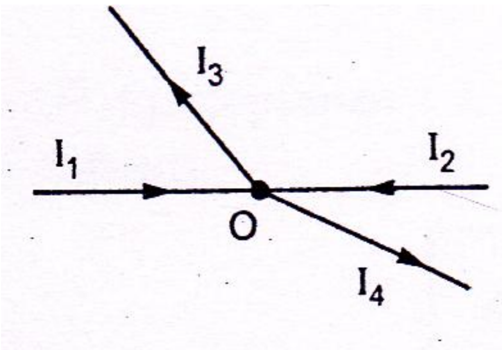

Fig. 1 Junction point

The law can be stated as,

The total current flowing

equal to the total current flowing away from

point.

- Another way to state the law is,
- The algebraic sum of all the current meeting at a junction point is always zero.
- The word algebraic means considering the signs of various currents.

$\sum I$ at junction point = 0

- Sign convention : Currents flowing towards a junction point are assumed to be positive whie currents flowing away from a junction point assumed to be negative.
- e.g. Refer to Fig. 1, currents I_1 and I_2 are positive while I_3 and I_4 are negative.
- Applying KCL, $\sum I$ at junction **0** = 0
- $I_1 + I_2 - I_3 - I_4 = 0$ i.e. $I_1 + I_2 = I_3 + I_4$
- The law is very helpful in network simplification.
- Kirchhoff's voltage law :

- " In any network, the algebraic sum of the voltage drops across the circuit elements of any closed path (or loop or mesh) is equal to the algebraic sum of the e.m.f s in the path"
- In other words, " the algebraic sum of all the branch voltages, around any closed path or closed loop is always zero."

Around a closed path $\sum V = 0$

- The law states that if one starts at a certain point of a closed path and goes on tracing and noting all the potential changes (either drops or rises), in any one particular direction, till the starting point reached again, he must be at the same potential with which he started tracing a closed path.
- Sum of all the potential rises must be equal to sum of all the potential drops while tracing any closed path of the circuit. The total change in potential along a closed path is always zero.
- This law is very useful in loop analysis of the network.
- A circuit consists of two parallel resistors having resistance of 20 Ω and 30 Ω respectively connected in series with 15Ω Ω resistor is 3 A,
- Find : i) Current in 20Ω and 30Ω resistors ii) The voltage across the whole circuit iii) The total power and power consumed in all resistors. (8)
- **Sol. :** The arrangement is shown in the Fig. 2.
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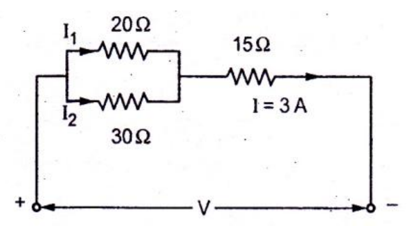


Fig. 2

Total current I = 3 A

$R_{eq} = (20 \parallel 30) + 15$

$= \frac{20 \times 30}{20 + 30} + 15$

$= 27\Omega$

$\therefore \quad I = \frac{V}{R_{eq}}$ i.e. $3 - \frac{V}{27}$

$\therefore \quad V = 81$ V … Voltage across each circuit

$I_1 = I \times \frac{30}{20+30} = 3 \times \frac{3}{5} = \mathbf{1.8\ A}$ … Current through 20Ω

$I_2 = I \times \frac{20}{20+30} = 3 \times \frac{2}{5} = \mathbf{1.2\ A}$ … Current through 30Ω

$P = V \times I = 81 \times 3 = \mathbf{243\ W}$ … Current total power

$P_{20\Omega} = I_1^2 \times 20 = (1.8)^2 \times 20 = \mathbf{64.8\ W}$

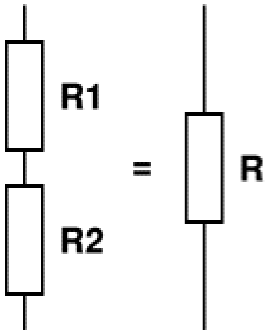
$P_{30\Omega} = I_2^2 \times 30 = (1.2)^2 \times 30 = \mathbf{43.2\ W}$

$P_{15\Omega} = I^2 \times 15 = (3)^2 \times 15 = \mathbf{135\ W}$

Cross check is $P = P_{20} + P_{30} + P_{15}$

Resistance

Resistance is the property of a component which **restricts the flow of electric current**. Energy is used up as the voltage across the component drives the current through it and this energy appears as heat in the component.



Resistors connected in Series

When resistors are connected in series their combined resistance is equal to the individual resistances added together. For example if resistors R1 and R2 are connected in series their combined resistance, R, is given by:

Combined resistance in **series**:

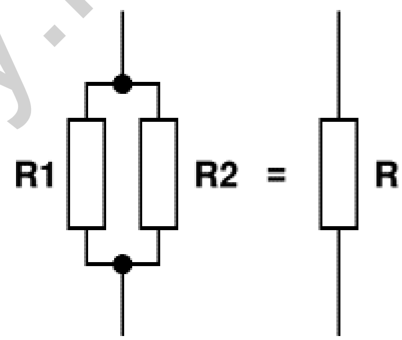
$$\mathbf{R = R1 + R2}$$

This can be extended for more resistors: **R = R1 + R2 + R3 + R4 + ...**

Note that the **combined resistance in series** will always be **greater** than any of the individual resistances.

Resistors connected in Parallel

When resistors are connected in parallel their combined resistance is less than any of the individual resistances. There is a special equation for the combined resistance of **two** resistors R1 and R2:



Combined resistance of **two resistors in parallel**: $\mathbf{R = \dfrac{R1 \times R2}{R1 + R2}}$

For more than two resistors connected in parallel a more difficult equation must be used. This adds up the **reciprocal** ("one over") of each resistance to give the **reciprocal** of the combined resistance, R:

$$\frac{1}{R} = \frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3} + \ldots$$

The simpler equation for **two** resistors in parallel is much easier to use!

Note that the **combined resistance in parallel** will always be **less** than any of the individual resistances.

(P) Power is the amount of current times the voltage level at a given point measured in wattage or watts.

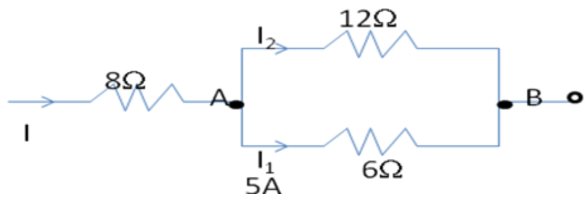
electrical energy - energy made available by the flow of electric charge through a conductor; "they built a car that runs on electricity" measured in k Watt Hour

Energy=VItKWhour

A 8 ohm resistor is in series with a parellel combination of two resistors 12 ohm and 6 ohm. If the current in the 6 ohm resistor is 5 A, determine the total power dissipated in the circuit. **(6)**

Ans. : The arrangement is shown in the fig.

$V_{AB} = I_1 \times 6 = 5 \times 6 = 30$ V



$\therefore I_2 = \frac{V_{AB}}{12} = \frac{30}{12} = 2.5$ A

$\therefore I = I_1 + I_2 = 7.5$ A

$\therefore$ $P_{12} = I_2^2 \times 12 = (2.5)^2 \times 12 = 75$ W

$\therefore$ $P_6 = I_1^2 \times 6 = (5)^2 \times 6 = 150$ W

$\therefore$ $P_8 = I^2 \times 8 = (7.5)^2 \times 8 = 450$ W

$\therefore$ $P_T = P_{12} + P_6 + P_8 = 675$ W.

2) A coil consists of 600 turns and a current of 10 A in the coil gives rise to a magnetic flux of 1 milli Weber. Calculate i) Self induction ; ii) The e.m.f. induced and iii) The energy stored when the current is reversed in 0.01 second . **(5)**

Ans.: N = 600, I = 10 A , Φ = 1 mWb

i) $L = N\Phi/I = \frac{600 \times 1 \times 10^{-3}}{10}$ = **0.06H**

ii) Current is reversed in 0.01 sec i.e. $I_2 = -10$ A, $I_1 = 10$ A

$\therefore$ $e = -L\frac{dI}{dt} = -L\frac{I_2 - I_1}{\Delta t} = \frac{-0.06 \times [-10 - 10]}{0.01}$ = **120v**

iii) $E = ½ LI^2 = ½ \times 0.06 \times (10)^2 = 3$ **J**

A current of 20 A flows through two ammeters A and B in series. The potential difference across A is 0.2 V and across B is 0.3 V.Find how the same current will divide between A and B when they are in parallel.

Ans : R_A = resistance of ammeter A

$= \frac{V_A}{1} = \frac{0.2}{20} = 0.01\Omega$

$\therefore$ R_B = resistance of ammeter B

$= \frac{V_B}{1} = \frac{0.3}{20} = 0.015\Omega$

Now the ammeters are connected in parallel as shown in the Fig. 1(b)

$R_{eq} = R_A \| R_B = \frac{0.01 \times 0.015}{0.01 + 0.015}$

$= 0.006\Omega$

$\therefore$ V = voltage across both the ammeters

∴ $V = I_A \times R_A = I_B \times R_B$

While $V = I \times R_{eq} = 20 \times 0.006 = 0.12$ V

∴ $I_A = \frac{V}{R_A} = \frac{0.12}{0.01} = \mathbf{12\ A}$

and $I_B = \frac{V}{R_B} = \frac{0.12}{0.01} = \mathbf{8\ A}$

Note: This can be verified using current division rule as,

$I_A = I \times \frac{R_B}{R_A + R_B} = \mathbf{12\ A}$ and $I_B = I \times \frac{R_A}{R_A + R_B} = \mathbf{8\ A}$

A parallel circuit comprises a resistor of 20 ohm in series with an inductive reactance of 15 ohm in one in one branch and a resistor of 30 ohm in series with a capacitive reactance of 20 ohm in the other branch. Determine the current and power dissipated in each branch if the total current drawn by the parallel circuit is $10\angle -30^0$ Amp**(8)**

Ans.: The arrangement is shown in the Fig.2.

$Z_1 = 20 + j\,15\ \Omega = 25 \angle 36.869^0\ \Omega$

$Z_2 = 30 - j\,20\ \Omega = 36.055 \angle -33.69^0\ \Omega$

By current division rule,

$I_1 = I_T \times \frac{Z_2}{Z_1 + Z_2} = \frac{10 \angle -30^0 \times 36.055 \angle 33.69^0}{20 + j15 + 30 - j20}$

$= \frac{360.055 \angle -63.90^0}{50.2493 \angle -5.71^0} = \mathbf{7.1752\angle -57.98^0\ A}$

And $I_2 = I_T \times \frac{Z_1}{Z_1 + Z_2} = \frac{10 \angle -30^0 \times 25 \angle 36.869^0}{50.2493 \angle -5.71^0} = \mathbf{4.9752 + 12.579^0\ A}$

Only resistive part of each branch consumes the power given by $(I)^2\,R$

∴ $P_1 = (I_1)^2 \times R_1 = (7.1752)^2 \times 20 = \mathbf{1029.67\ W.}$

<u>Faraday's Laws:</u>

<u>1st law</u>: Whenever magnetic flux linking with a coil changes with time an emf is induced in that coil or whenever a moving conductor cuts the magnetic flux, an emf is induced in the conductor.

<u>2nd law:</u> The magnitude of the induced emf is equal to the product of the number of turns of the coil and the rate of change of flux linkage.

Lenz's law :

It states that the direction of an induced emf produced by the electromagnetic induction is such that it sets up a current which always opposes the cause that is responsible for inducing the emf.

In short the induced emf always opposes the cause producing it which is represented by negative sign, mathematically in its expression

Consider a solenoid as shown in Fig.1. Let a bar magnet is moved towards coil such that N-pole of magnet is facing a coil which will circulate the current through the coil.

According to Lenz's law, the direction of current due to induced emf is so as to oppose the cause. The cause is motion of bar magnet towards coil So emf will set up a current through coil in such a way that the end of solenoid facing bar magnet will become N-pole. Hence two like poles will face each other experiencing force of repulsion which is opposite to the motion of bar magnet as shown in the above .

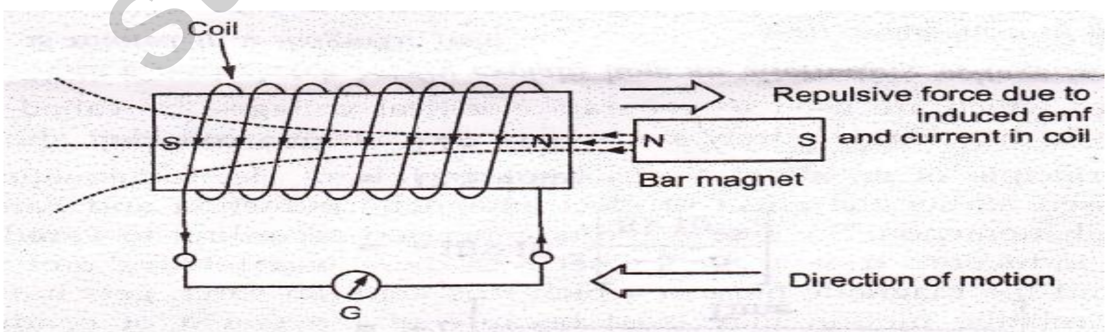
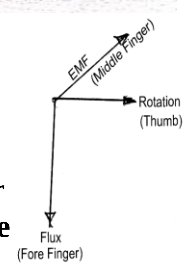


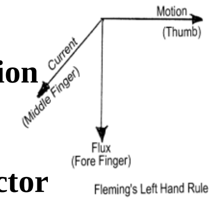
Fig. 1

1. <u>Fleming's Right hand rule</u>: **This rule helps in deciding the direction of the induced emf.**
   - **Hold the right hand thumb, fore finger and the middle finger set at right angles to each other and the *thumb* points the direction of the *motion* of the conductor and the *fore finger* points the direction of the *field* and the *middle finger* points the direction of the induced *emf*.**



Fleming's Right Hand Rule

2. Fleming's Left hand rule: **This rule helps in deciding the direction of force acting on a conductor.**
   - **Hold the left hand thumb, fore finger and the middle finger set at right angles to each other and the** *thumb* **points the direction of the** *force* **acting on the conductor and the direction of the** *fore finger* **points the direction of the** *magnetic field* **and the** *middle finger* **points the direction of the** *current* **in the conductor**



Statically and dynamically induced emf's
Statically induced emf **STATICALLY INDUCED EMF**

The emf induced in a coil due to change of flux linked with it (change of flux is by the increase or decrease in current) is called statically induced emf.

Transformer is an example of statically induced emf. Here the windings are stationary,magnetic field is moving around the conductor and produces the emf.

DYNAMICALLY INDUCED EMF

The emf induced in a coil due to relative motion of the conductor and the magnetic field is called dynamically induced emf.

example:dc generator works on the principle of dynamically induced emf in the conductors which are housed in a revolving armature lying within magnetic field

Statically induced e.m.f

The change in flux lines with respect to coil can be achieved without physically moving the coil or the magnet. Such induced e.m.f. in a coil which is without physical movement of coil or a magnet is called **statically induced e.m.f.**

To have an induced e.m.f there must be change in flux associated with a coil. Such a change in flux can be achieved without any physical movement by increasing and decreasing the current producing the flux rapidly, with time.

Consider an electromagnet which is producing the necessary flux for producing e.m.f. Now let current through the coil of an electromagnet be an alternating one. Such alternating current means **it changes its magnitude periodically with time**. This produces the flux which is also alternating i.e. changing with time. Thus there exists $\frac{d\phi}{dt}$ associated with coil placed in the viscinity of an electromagnet. This is responsible for producing an e.m.f in the coil. This is called statically induced e.m.f.

There is no physical movement of magnet or conductor; it is the alternating supply which is responsible for such an induced e.m.f.

Dynamically induced e.m.f.

The change in the flux linking with a coil, conductor or circuit can be brought about by its motion relative to magnetic field. This is possible by moving flux with respect to coil conductor or circuit or it is possible by moving conductor, coil, circuit with respect to stationary magnetic flux.

Such an induced e.m.f. which is due to **physical movement** of coil, conductor with respect to flux or movement of magnet with respect with to stationary coil, conductor is called **dynamically induced e.m.f. or motional induced e.m.f.**

This type of induced e.m.f. is available in the rotating machines such as alternators, generator etc.

d) A coil resistance 150Ω is placed in a magnetic field of 0.1 mWb. The coil has 500 turns and a galvanometer of 450Ω is connected in series with it. The coil is moved in 0.1 sec from the given field to another field of 0.3 mWbs. Find the average induced e.m.f. and the average current through the coil. **[5]**

Ans : N = 500, ϕ_2 = 0.3 mWb, ϕ_1 = 0.1mWb, dt = 0.1 sec.

$$|e| = N\frac{d\phi}{dt} = \frac{N[\phi_2 - \phi_1]}{dt} = \frac{500\times[0.3-0.1]\times10^{-3}}{0.1}$$

$$= \mathbf{1\ V}$$

$$R_T = R_{coil} + R_g = 150+450 = 600\Omega$$

$$\therefore \quad I = \frac{|e|}{R_T} = \frac{1}{600} = 1.6667 \text{ mA.}$$

Find the currents in all the resistors of the network shown in Fig. 1. Also find the voltage across AB. **[6]**

s.: The current distribution in various branches is shown in the Fig. 2.

Apply KVL to the two loops,

Loop ACDA,

$$-10 I_1 - 5 I_2 + 20(1-I_1) = 0$$

$$30I_1 + 5I_2 = 20 \quad \ldots (1)$$

Loop CBDC,

$$-2.5(I_1-I_2) + 5(1-I_1+I_2) + 5I2 = 0$$

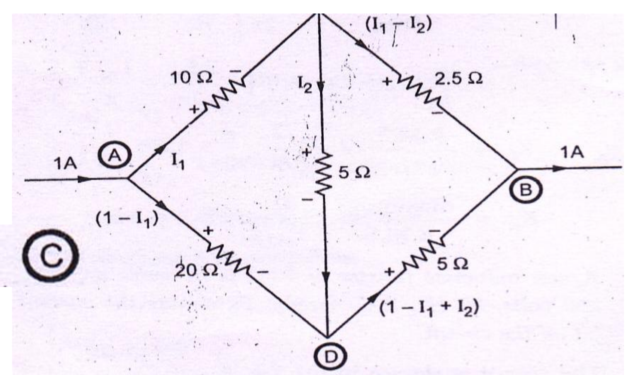


Fig. 2

$$\therefore \quad 7.5I_1 - 12.5I_2 = 5 \quad \ldots (2)$$

Solving (1) and (2) , I_1 = 0.666 A, I_2 = 0 A

Hence the currents in all the resistors are,

Branch	Resistance	Current
AC	10Ω	0.666 A
AD	20Ω	0.333 A
CD	5Ω	0 A
CB	205Ω	0.666 A
DB	5Ω	0.333A

Trace the path from A to B as shown in the Fig. 3.

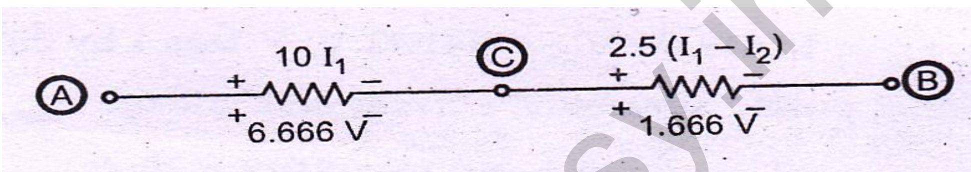


Fig. 3

∴ V_{AB} = 6.666+1.666
= **8.3333V with A positive**

With examples, clearly differentiate between statically induced e.m.f. and dynamically induced e.m.f.

Ans : **Statically induced e.m.f**

The change in flux lines with respect to coil can be achieved without physically moving the coil or the magnet. Such induced e.m.f. in a coil which is without physical movement of coil or a magnet is called **statically induced e.m.f.**

To have an induced e.m.f there must be change in flux associated with a coil. Such a change in flux can be achieved without any physical movement by increasing and decreasing the current producing the flux rapidly, with time.

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There is no physical movement of magnet or conductor; it is the alternating supply which is responsible for such an induced e.m.f.

Such type of an induced e.m.f. is available in transformers.

Dynamically induced e.m.f.

The change in the flux linking with a coil, conductor or circuit can be brought about by its motion relative to magnetic field. This is possible by moving flux with respect to coil conductor or circuit or it is possible by moving conductor, coil, circuit with respect to stationary magnetic flux.

Such an induced e.m.f. which is due to **physical movement** of coil, conductor with respect to flux or movement of magnet with respect with to stationary coil, conductor is called **dynamically induced e.m.f. or motional induced e.m.f.**

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[5]

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$$R_T = R_{coil} + R_g = 150+450 = 600\Omega$$

$$\therefore \quad I = \frac{|e|}{R_T} = \frac{1}{600} = 1.6667 \text{ mA.}$$

self inductance : According to Lenz's law the direction of this induced e.m.f. will be so as to oppose the cause producing it. The cause is the current I hence self induced e.m.f will try to set up a current which is in opposite direction to that of current I. When current is increased, self induced e.m.f. reduces the current tries to keep to its original value. If current is decreased, self induced e.m.f. increases the current and tries to maintain it back to its original value. So any change in current through coil is opposed by the coil.

This property of the coil which opposes any change in the current passing through it is called self inductance or only inductance.

It is analogous to electrical inertia or electromagnetic inertia.

The formula for self inductance is given by,

$$L = \frac{N\Phi}{I}$$

It can be defined as flux linkages per ampere current in it. Its unit is Henry (H)

Expressions for coefficient of self inductance (L):

$$L = \frac{N\Phi}{I}$$

But

$$\Phi = \frac{mmf}{reluctance}$$

$$= \frac{NI}{S}$$

∴ $L = \frac{N.NI}{I.S}$

∴ $L = \frac{N^2}{S}$ henries

Now $s = \frac{1}{\mu a}$

$L = \frac{N^2}{\left(\frac{1}{\mu a}\right)}$

∴ $L = \frac{N^2 \mu a}{l} = \frac{N^2 \mu_0 \mu_r a}{l}$ Henries

Where l = length of magnetic circuit
a = area of cross-section of magnetic circuit which flux is passing.

Derive an Expression for energy stored in the inductor:
Let the induced e.m.f. in a coil be,

$e = -L\frac{dI}{dt}$

This opposes a supply voltage. So supply voltage 'V' supplies energy to overcome this, which ultimately gets stored in the magnetic field.

∴ $V = -e = -\left[-L\frac{dI}{dt}\right] = L\frac{dI}{dt}$

∴ Power supplied = V × I = $L\frac{dI}{dt}$ × I

∴ Energy supplied in time dt is,

E = power x time = $L\frac{dI}{dt}$ x I x dt

= L di x I joules.

This is energy supplied for a change in current of dI but actually current changes from zero to I.

∴ Integrating above total energy stored is,

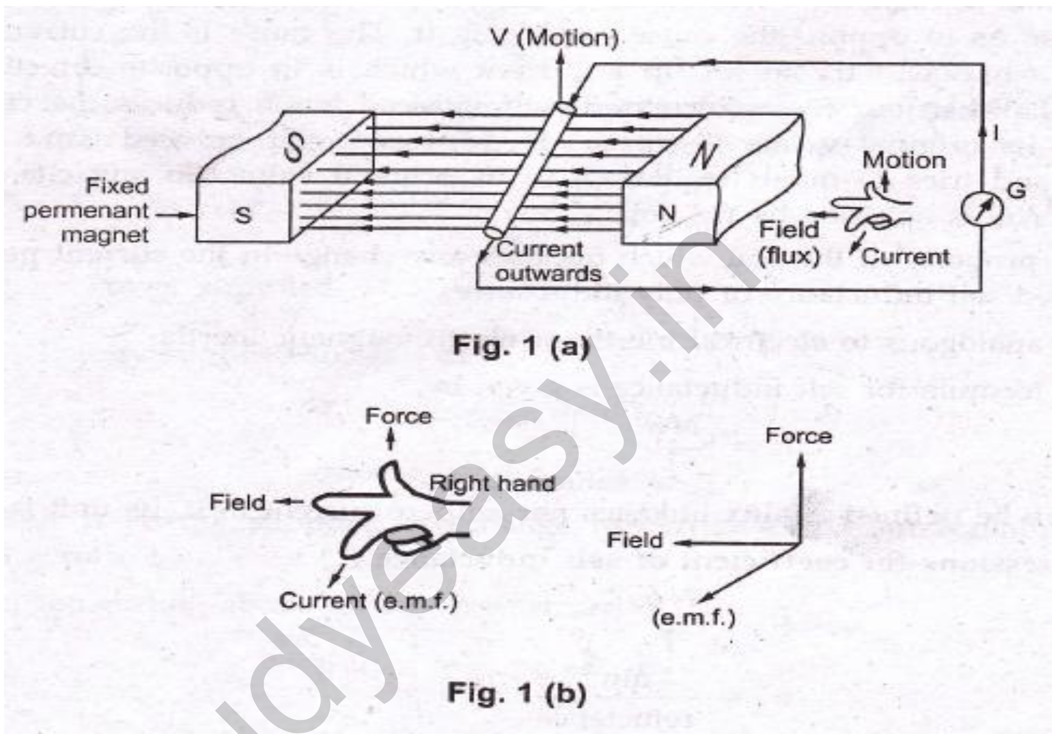
E =

$$\int_0^I L dI \;\; I = L\int_0^I dI \;\; I$$

state i) Flemming's right hand rule, and
ii) Fleming's left hand rule.
Mention their applications.

Fleming's right hand rule : The Fleming's left hand rule is used to get direction of force experienced by conductor carrying current placed in magnetic field while Fleming's right hand rule can be used to get direction of induced e.m.f. when conductor is moving at right angles to the magnetic field.

According to this rule, outstretch the three fingers of right hand namely the thumb, fore finger and the middle finger, perpendicular to each other. Arrange the right hand so that finger point in the direction of flux lines (from N to S) and thumb in the direction of motion of conductor with respect to the flux then the middle finger will point in the direction of the induced e.m.f. (or current).



(1)

Fleming's left hand rule: The direction of the force experienced by the current carrying conductor placed in magnetic field can be determined by a rule called 'Fleming's left hand rule'. The rule states that ' outstretch the three fingers on the left hand namely the first finger, middle finger and thumb such that they are mutually perpendicular to each other. Now point the first finger in the direction of magnetic field and middle finger in the direction of the current then the thumb gives the direction of the force experienced by the conductor.'

The rule is explained in the diagrammatic form in fig. 2.

Applications: Fleming's right hand rule is used to get the direction of induced emf in case of generators and alternators while left hand rule is used to get the direction of torque induced in motors.

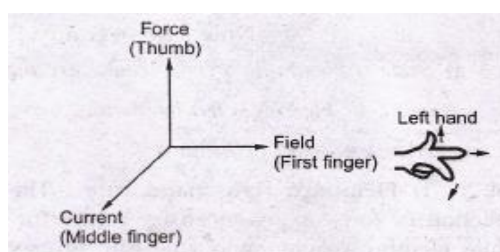


Fig. 2 Fleming's left hand rule

Define i) self inductance, and ii) mutual inductance.

Mention their units and formula to calculate each of them. Derive an expression for the energy stored in an inductor of self inductance 'L' Henry carrying the current of 'I' amperes.

Sol. : self inductance : According to Lenz's law the direction of this induced e.m.f. will be so as to oppose the cause producing it. The cause is the current I hence self induced e.m.f will try to set up a current which is in opposite direction to that of current I. When current is increased, self induced e.m.f. reduces the current tries to keep to its original value. If current is decreased, self induced e.m.f. increases the current and tries to maintain it back to its original value. So any change in current through coil is opposed by the coil.

This property of the coil which opposes any change in the current passing through it is called self inductance or only inductance.

It is analogous to electrical inertia or electromagnetic inertia.

The formula for self inductance is given by,

$$L = \frac{N\Phi}{I}$$

It can be defined as flux linkages per ampere current in it. Its unit is Henry (H)

Expressions for coefficient of self inductance (L):

$$L = \frac{N\Phi}{I}$$

But $$\Phi = \frac{mmf}{reluctance}$$

$$= \frac{NI}{S}$$

$$\therefore \quad L = \frac{N \cdot NI}{I \cdot S}$$

$$\therefore \quad L = \frac{N^2}{S} \text{ henries}$$

Now $$s = \frac{l}{\mu a}$$

$$L = \frac{N^2}{\left(\frac{l}{\mu a}\right)}$$

$$\therefore \quad L = \frac{N^2 \mu a}{l} = \frac{N^2 \mu_0 \mu_r a}{l} \text{ Henries}$$

Where l = length of magnetic circuit

a = area of cross-section of magnetic circuit through which flux is passing.

8) Derive an Expression for energy stored in the inductor:

Let the induced e.m.f. in a coil be,

$$e = -L\frac{dI}{dt}$$

This opposes a supply voltage. So supply voltage 'V' supplies energy to overcome this, which ultimately gets stored in the magnetic field.

$$\therefore \quad V = -e = -\left[-L\frac{dI}{dt}\right] = L\frac{dI}{dt}$$

$\therefore$ Power supplied = V × I = L $\frac{dI}{dt}$ × I

$\therefore$ Energy supplied in time dt is,

E = power x time = L $\frac{dI}{dt}$ x I x dt

= L di x I joules.

This is energy supplied for a change in current of dI but actually current changes from zero to I.

$\therefore$ Integrating above total energy stored is,

$$E = \int_0^I L\,dI\ \ I = L\int_0^I dI\ \ I$$

$$= L\left[\frac{I^2}{2}\right]_0^I = L\left[\frac{I^2}{2} - 0\right]$$

$$E = \frac{1}{2} L I^2 \text{ joules}$$

Mutual inductance:

Magnitude of mutually induced e.m.f

Let N_1 = Number of turns of coil A

N_2 = Number of turns of coil B

I_1 = Current flowing through coil A

$\emptyset_1$ = Flux produced due to current I_1 in webers.

$\emptyset_2$ = Flux linking with coil B

According to Faraday's law, the induced e.m.f. in coil B is,

$$e_2 = -N_2 \frac{d\emptyset_2}{dt}$$

Negative sign indicates that this e.m.f will set up a current which will oppose the change of flux linking with it.

Now $$\emptyset_2 = \frac{\emptyset_2}{I2} \times I_1$$

If permeability of the surroundings is assumed constant then $\emptyset_2 \propto I_1$ and hence $\emptyset/I_1$ is constant.

$\therefore$ Rate of change of $\emptyset_2 = \frac{\emptyset_2}{I_1} \times$ Rate of change of current I_1

$$\therefore \quad \frac{d\emptyset_2}{dt} = \frac{\emptyset_2}{I_1} \cdot \frac{dI_1}{dt}$$

$$\therefore \quad e_2 = -N_2 \cdot \frac{\emptyset_2}{I_1} \cdot \frac{dI_1}{dt}$$

$\therefore \qquad e_2 = -\left(\frac{N_2 \phi_2}{I_1}\right) \frac{dI_2}{dt}$

Here $\left(\frac{N_2 \phi_2}{I_1}\right)$ is called co efficient of mutual inductance dented by M

$$e_2 = -M\frac{dI_1}{dt} \quad \text{Volts}$$

Coefficient of mutual inductance is defined as the property by which e.m.f gets induced in the second coil because of change in current through first coil.

Coefficient of mutual inductance is defined as the property by which e.m.f gets induced in the second coil because of change in current through first coil.

Coefficient of mutual inductance is also called mutual inductance. It is measured in Henries.

Definitions of mutual inductance and its unit:

1) The coefficient of mutual inductance is defined as the flux linkages of the coil per ampere current in other coil.
2) It can also be defined as equal to e.m.f induced in volts in one coil when current in other coil changes uniformly are rate of one ampere per second.

Similarly its unit be defined as follows:

1) Two coils which are magnetically coupled are said to have mutual inductance of one hence when a current of one ampere flowing through one coil produces a flux linkage of one Weber turn in the other coil.
2) Two coils which are magnetically coupled are said to have mutual inductance of one Henry when a current changing uniformly at the rate of one ampere per second in one coil, induces as e.m.f of one volts in the other coil.

9) Expressions of the mutual inductance (M):

1) $$M = \frac{N_2 \phi_2}{I_1}$$

2) ϕ_2 is the part of the flux ϕ_1 produced due to I_1. Let K_1 be the fraction of ϕ_1 which is linkage with coil B.

$$\therefore \quad \phi_2 = K_1 \phi_1$$

$$M = \frac{N_2 K_1 \phi_1}{I_1}$$

3) The flux ϕ_1 can be expressed as,

$$\phi_1 = \frac{\text{m.m.f}}{\text{reluctance}} = \frac{N_1 I_1}{S}$$

$$M = \frac{N_2 K_1}{I_1}\left(\frac{N_1 I_1}{S}\right)$$

$$M = \frac{K_1 N_1 N_2}{S}$$

If all the flux produced by the coil A links with coil B K_1= 1.

$$M = \frac{N_1 N_2}{S}$$

$$s = \frac{l}{\mu a} \text{ and } K_1 = 1$$

Then $$M = \frac{N_1 N_2}{\left(\frac{l}{\mu a}\right)} = \frac{N_1 N_2 a\mu}{l}$$

$$M = \frac{N_1 N_2 a \mu_0 \mu_r}{l}$$

5) If second coil carries current I_2, producing flux $\emptyset_2$, the part of which links with coil A i.e.$\emptyset_1$ then,

$$\emptyset_1 = K_2\, \emptyset_2 \quad \text{and}$$

$$M = \frac{N_1 \emptyset_2}{I_2}$$

$$M = \frac{N_1 K_2 \emptyset_2}{I_2}$$

Now $$\emptyset_2 = \frac{N_2 I_2}{S}$$

$$M = \frac{N_1 K_2 N_2 I_2}{I_2 S}$$

$$M = \frac{N_1 K_2 N_2}{S}$$

coupling coefficient: The coefficient of coupling is define as the ratio of the actual mutual inductance present between the two coils as the maximum possible value of the mutual inductance. It gives an idea about magnetic coupling between the two coils. This coefficient indicates the amount of linking with other coil which is produced by one coil.

Let N_1 = Number of turns of first coil
N_2 = number of turns of second coil
I_1 = current through first coil
I_2 = current through by first coil
$_1$ = flux produced by first coil
ϕ_2 = flux produced by second coil
ϕ

$\therefore \quad M = \frac{N_2 K_1 \phi_1}{I_1}$ and $M = \frac{N_1 K_2 \phi_2}{I_2}$

Multiplying the two expressions,

$$M \times M = \frac{N_2 K_1 \phi_1}{I_1} \times \frac{N_1 K_2 \phi_2}{I_2}$$

$$M_2 = K_1 K_2 \left[\frac{N_1 \phi_1}{I_1}\right] \left[\frac{N_2 \phi_2}{I_2}\right]$$

But $\frac{N_1 \phi_1}{I_1} = L_1$ = self inductance of first coil

And $\frac{N_2 \phi_2}{I_2} = L_2$ = self inductance of second coil

$\therefore \quad M_2 = K_1 K_2 L_1 L_2$

$\therefore \quad M = \sqrt{K_1 K_2} \quad \sqrt{L_1 L_2}$

Let $\quad K = \sqrt{K_1 K_2}$ = coefficient of coupling

$\therefore \quad M = K\sqrt{L_1 L_2}$

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

Three similar coils each having resistance of 10 ohm and reactance of 8 ohm are connected in star across a 400 V, 3 phase supply . Determine the i) Line current; ii) Total power and iii) Reading of each of two wattmeters connected to measure the power. **(8)**

Ans.: R=10 Ω , $X_L = 8\ \Omega$, $V_L = 400$ V, star

$\therefore \quad Z_{ph} = 10 + j\,8\ \Omega = 12.082 \angle 38.659^0\ \Omega$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94$ V,

…star

i) $\quad I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{12.8062} = 18.0334$ V,

$\therefore \quad I_L = I_{ph} = 18.0334$ A.

ii) $\quad P_T = \sqrt{3}\ V_L\ I_L \cos\Phi$ where $\Phi = 38.659^0$

$= \sqrt{3} \times 400 \times 18.0334 \times \cos 38.659^0$

$= 9756.2116$ W

iii) $\quad W_1 = V_L I_L \cos(30 - \Phi) = 400 \times 18.0334 \times \cos(30^0 - 38.659^0)$

= 7131.1412 W

$W_2 = V_L I_L \cos(30 + \Phi) = 400 \times 18.0334 \times \cos(30^0 + 38.659^0)$

= 2625.0704 W

A coil of 300 turns wound on a core of non magnetic material has an inductance of 10 mH. Calculate i) flux produced by a current of 5A ii) the average value of the emf induced when a current of 5Amps is reversed in 8 milli seconds.

Sol: Given values are N = 300, I = 5A, L = 10mH = 10×10^{-3}H

Self inductance, $L = \frac{N\phi}{I}$ $\therefore \phi = \frac{LI}{N} = \frac{10x10^{-3}x5}{300}$

$\therefore$ $\phi = 0.166mWb$

Self induced emf, $e = -L\frac{di}{dt}$

$= -L\left[\frac{Final\ current - intial\ current}{Time}\right]$

$= -10x10^{-3}\left[\frac{-5-5}{8 \times 10^{-3}}\right]$

$= 12.5$ volts

$\therefore$ Induced emf = 12.5 volts

A coil of 300 turns wound on a core of non magnetic material has an inductance of 10 mH.Calculate i) flux produced by a current of 5A ii) the average value of the emf induced when a current of 5Amps is reversed in 8 milli seconds.

Sol: Given values are N = 300, I = 5A, L = 10mH = $10x10^{-3}$H

Self inductance, $L = \frac{N\phi}{I}$ $\phi = \frac{LI}{N} = \frac{10x10^{-3}x5}{300}$

$\phi = 0.166mWb$ $\therefore$

$\therefore$

Self induced emf, $e = -L\frac{di}{dt}$

$= -L\left[\frac{Final\ current - intial\ current}{Time}\right]$

$= -10x10^{-3}\left[\frac{-5-5}{8 \times 10^{-3}}\right]$

$= 12.5$ volts

$\therefore$ Induced emf = 12.5 volts

Two coupled coils of self inductances 0.8 H and 0.20 H have a coefficient of coupling 0.9. Find the mutual indulgence and turns ratio [7]

Sol.: $L_1 = 0.8$ H

$L_2 = 0.2$ H

$K = 0.9$

Mutual Inductance, $M = K\sqrt{L_1 L_2} = 0.9\sqrt{(0.8)(0.2)}$

$= 0.9\sqrt{0.16} = (0.9)(0.4) = 0.36$ H

$\therefore$ M = 0.36 H

Turns ratio $\frac{L_1}{L_2} = \frac{N_1}{N_2}$

$\therefore$ $\frac{N_1}{L_2} = 0.\frac{8}{0}.2 = 4$

$\therefore$ Turns ratio = 4

A coil of 1000 turns is wound on a silicon steel ring of relative permeability 1200. The ring has a mean diameter of 10cm and cross sectional area of 12 sq.cm. When a current of 4 amperes flows through the coil. Find

i) Flux in the core

ii) Inductance of the coil

iii) The emf induced in the coil if the flux falls to zero in 15 milii seconds.

iv) Now, if another similar coil is placed such that 70% magnetic coupling exists between the coils. Find the mutual inductance. (Chapter-1) [10]

Ans.: Given:

N = 1000 turns

$\mu_r = 1200$

d = 10 cm = 0.1 m

a = 12 cm^2 = 12×$10^{-4} m^2$

I = 4 A

$\mu_o = 4\pi \times 10^{-7}$

i) Flux in the core:

Let I = Length of magnetic circuit

= $\pi \times d_{mean}$

= $\pi \times 0.1$ m

= 0.3141 m

$$S = \frac{I}{\mu_0 \mu_{r}}$$

$$= \frac{0.3141}{4\pi \times 10^{-7} \times 1200 \times 12 \times 10^{-4}}$$

= 1.735 × 10^5 AT/Wb

$$\emptyset = \frac{NI}{S}$$

$$= \frac{1000 \times 4}{1.735 \times 10^5}$$

10^{-3} Wb

∴ flux in the core = ∅ = **23.05 × 10^{-3} Wb**

ii) Inductance of the coil:

$$L = \frac{N^2}{S} = \frac{(1000)^2}{1.735 \times 10^5}$$

L = **5.763 H**

iii) EMF induced in the coil:

$$= -N\frac{d\emptyset}{dt}$$

$$= -1000 \times \frac{(23.05 \times 10^{-3} - 0)}{15 \times 10^{-3}}$$

= 1536.6V

iv) 70% of magnetic coupling i.e.

$$K_1 = \frac{\emptyset_2}{\emptyset_1} = 0.7$$

Mutual Inductance =

$$M = \frac{N\emptyset_3}{I}$$

$$= \frac{1000 \times 0.7\emptyset_1}{4}$$

$$\frac{1000 \times 0.7 \times 23.05 \times 10^{-3}}{4}$$

= 4.033 H

Unit-II

Single-phase A.C. Circuits: Generation of sinusoidal voltage, definition of average value, root mean square value, form factor and peak factor of sinusoidally varying voltage and current, phasor representation of alternating quantities. Analysis, with phasor diagrams, of R, L, C, R-L, R-C and R-L-C circuits, real power, reactive power, apparent power and power factor. Illustrative examples involving series, parallel and series-parallel circuits

SINGLE-PHASE CIRCUITS

3.1 Generation of sinusoidal AC Voltage

Alternating voltage may be generated:

a) By rotating a coil in a magnetic field as shown in Fig.3.1.

b) By rotating a magnetic field within a stationary coil as shown in Fig.3.2.

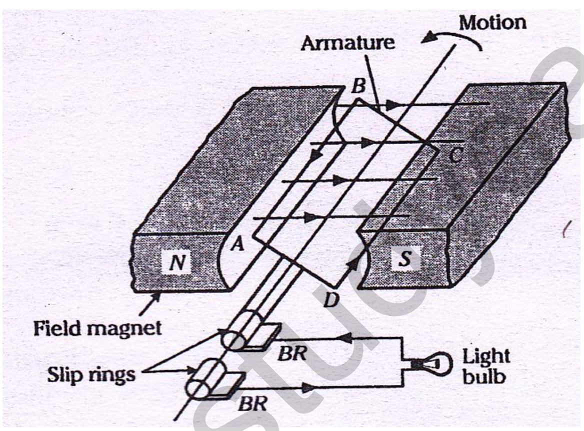


Fig. 3.1

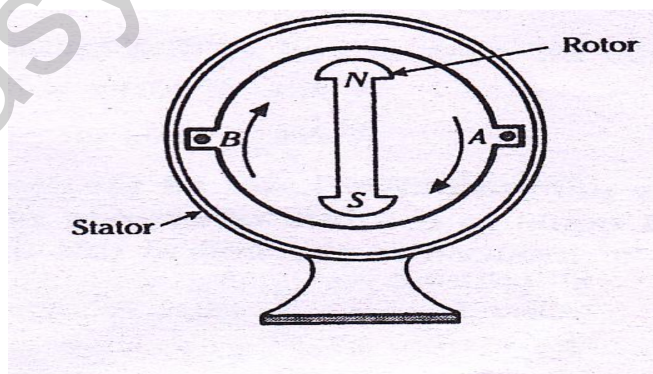


Fig. 3.2

" In each case, the value of the alternating voltage generated depends upon the number of turns in the coil, the strength of the field and the speed at which the coil or magnetic field ritates."

The alternating voltage generated has regular changes in magnitude and direction. If a load resistance (e.g. a light bulb) is connected across this alternating voltage, an alternating current flows in the circuit. When there is a reversal of polarity of the alternating voltage, the direction of current flow in the circuit also reverses.

3.2 Equation of Alternating E.M.F.

Let us take up the case of a rectangular coil of N turns rotating in the anticlockwise direction, with an angular velocity of ω radians per second in a uniform magnetic field as shown in Fig.3.3. let the time be measured from the instant of coincidence of the plane of the coil with the

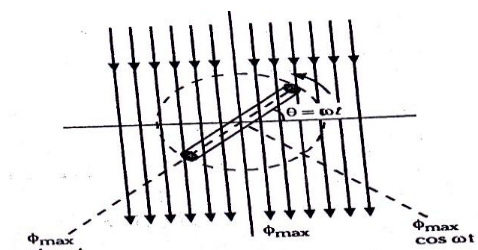


Fig. 3.3

X-axis. At this instant maximum flux ϕ_{max}, links with the coil. As the coil rotates, the flux linking with it changes and hence e.m.f. is induced in it. Let the coil turn through an angle θ in time 't' seconds, and let it assume the position as shown in Fig.3.3. Obviously $\theta = \omega t$.

When the coil is in this position, the maximum flux acting vertically downwards can be resolved into two components, each perpendicular to the other, namely:

a) Component $\phi_{max} \sin \omega t$, parallel to the plane of the coil. This component does not induce e.m.f. as it is parallel to the plane of the coil.
b) Component $\phi_{max} \cos \omega t$, perpendicular to the plane of coil. This component induces e.m.f. in the coil.

∴ flux linkages of coil at that instant (at θ^0) is

= No. of turns x flux linking

$= N \phi_{max} \cos \omega t$

As per faraday's Laws of Electromagnetic induction, the e.m.f. induced in a coil is equal to the rate of change of flux linkages of the coil. So, instantaneous e.m.f. 'e' induced in the coil at this instant is:

$$e = -\frac{d}{dt}(\text{flux linkages})$$
$$= -\frac{d}{dt}(N \phi_{max} \cos \omega t)$$
$$= -N \phi_{max} \frac{d}{dt}(\cos \omega t)$$
$$= -N \phi_{max}\, \omega\, (-\sin \omega t)$$
$$\therefore \quad e = +N \phi_{max} \sin \omega t \text{ volts} \quad \ldots(1)$$

It is apparent from eqn.(1) that the value of 'e' will be maximum (E_m), when the coil has rotated through 90^0 (as $\sin 90^0 = 1$)

$$\text{Thus} \quad E_m = N\, \omega\, \phi_{max} \text{ volts} \quad \ldots(2)$$

Substituting the value of $N\, \omega\, \phi_{max}$ from eqn.(2) in eqn.(1), we obtain:

$$e = E_m \sin \omega t \quad \ldots(3)$$

we know that $\theta = \omega t$

$$\therefore e = E_m \sin \theta$$

It is clear from this expression of alternating e.m.f. induced in the coil that instantaneous e.m.f. varies as the sin of the time angle (θ or ωt).

$\omega = 2\pi f$, where 'f' is the frequency of rotation of the coil. Hence eqn.(3) can be written as

$$e = E_m \sin 2\pi f t \quad \ldots(4)$$

If T = time of the alternating voltage = $\frac{1}{f}$, then eqn.(iv) may be re-written as

$$e = E_m \sin\left(\frac{2\pi}{T}\right)t$$

so, the e.m.f. induced varies as the sine function of the time angle, ωt, and if e.m.f. induced is plotted against time, a curve of sine wave shape is obtained as shown in Fig.3.4. Such an e.m.f. is called sinusoidal

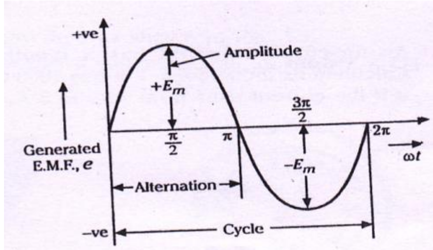


Fig. 3.4

when the coil moves through an angle of 2π radians.

3.3 Equation of Alternating Current

When an alternating voltage $e = E_m \sin \omega t$ is applied across a load, alternating current flows through the circuit which will also have a sinusoidal variation. The expression for the alternating current is given by:

$$i = I_m \sin \omega t$$

in this case the load is resistive (we shall see, later on, that if the load is inductive or capacitive, this current-equation is changed in time angle).

3.4 Important Definitions

Important terms/definitions, which are frequently used while dealing with a.c. circuits, are as given below:

1. Alternating quantity: An alternating quantity is one which acts in alternate positive and negative directions, whose magnitude undergoes a definite series of changes in definite intervals of time and in which the sequence of changes while negative is identical with the sequence of changes while positive.
2. Waveform: " The graph between an alternating quantity (voltage or current) and time is called waveform", Generally, alternating quantity is depicted along the Y-axis and time along the X-axis.fig.4.4 shows the waveform of a sinusoidal voltage.
3. Instantaneous value: The value of an alternating quantity at any instant is called instantaneous value.
   The instantaneous values of alternating voltages and current are represented by 'e' and 'I' respectively.
4. Alternation and cycle: When an alternating quantity goes through one half cycle (complete set of +ve or –ve values) it completes an alternation, and when it goes through a complete set of +ve and –ve values, it is said to have completed one cycle.
5. Periodic Time and Frequency: The time taken in seconds by an alternating quantity to complete one cycle is know as periodic time and is denoted by T.
   The number of cycles completed per second by an alternating quantity is know as frequency and is denoted by 'f'. in the SI system, the frequency is expressed in hertz.

   The number of cycles completed per second = f.

   Periodic Time T – Time taken in completing one cycle = $\frac{1}{f}$

   $$\text{Or } f = \frac{1}{T}$$

   In India, the standard frequency for power supply is 50 Hz. It means that alternating voltage or current completes 50 cycles in one second.
6. Amplitude: The maximum value, positive or negative, which an alternating quantity attains during one complete cycle is called amplitude or peak value or maximum

value. The amplitude of alternating voltage and current is represented by E_m and I_m respectively.

3.5 Different Forms of E.M.F. Equation

The standard form of an alternating voltage, as already mentioned in sec.3.2 is

$$e = Em \sin \theta = Em \sin \omega t = Em \sin 2\pi f\, t = Em \sin \frac{2\pi}{T} t$$

on perusal of the above equations, we find that

a) The amplitude or peak value or maximum value of an alternating voltage is given by the coefficient of the sine of the time angle.

b) The frequency 'f' is given by the coefficient of time divided by 2π.

Taking an example, if the equation is of an alternating voltages is given by e = 20 sin 314t, then its maximum value is 20 V and its frequency is

$$f = \frac{314}{2\pi}\ 50\ \text{Hz}$$

In a like manner, if the equation is of the form

$e = I_m\sqrt{(R^2 + 4\omega^2 L^2)} \sin 2\,\omega t$, then its maximum value is

$E_m = I_m \sqrt{(R^2 + 4\omega^2 L^2)}$ and the frequency is

$\frac{2\omega}{2\pi}$ or $\frac{\omega}{\pi}$ Hertz

3.6 Root-mean-square (R.M.S.) Value

The r.m.s. or effective value, of an alternating current is defined as that steady current which when flowing through a given resistance for a given time produces the same amount of heat as produced by the alternating current, when flowing through the same resistance for the same time.

Let us take two circuits with identical resistance, but one is connected to a battery and the other to a sinusoidal voltage source. Wattmeters are employed to measure heat power in each circuit. The voltage applied to each circuit is so adjusted that the heat power produced in each circuit is the same. In this event the direct current I will equal $\frac{I_m}{\sqrt{2}}$, which is termed r.m.s. value of the sinusoidal current.

The following method is used for finding the r.m.s. or effective value of sinusoidal waves.

The equation of an alternating current varying sinusoidally is given by

$$i = I_m \sin \theta$$

let us consider an elementary strip of thickness dθ in the first cycle of the squared wave, as shown in Fig.3.5. let i^2 be mid-ordinate of this strip.

Area of the strip = i2 d θ

Area of first half-cycle of squared wave

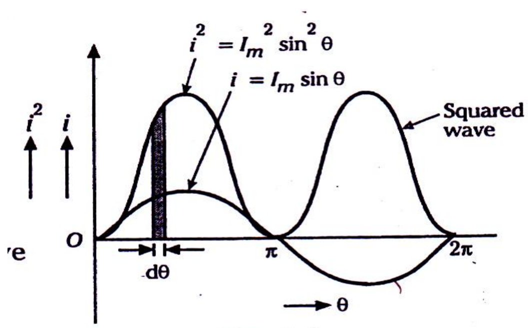


Fig. 3.5

$= \int_0^{\pi} i^2 \, d\theta$

$= \int_0^{\pi} (I_m \sin\theta)^2 \, d\theta$ $\qquad (\because I = I_m \sin\theta)$

$= \int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$

$= I_m^2 \int_0^{\pi} \frac{1-\cos 2\theta}{2} \, d\theta$ $\qquad (\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$

$= \frac{I_m^2}{2} \int_0^{\pi} (1 - \cos 2\theta) d\theta$

$= \frac{I_m^2}{2} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$

$= \frac{I_m^2}{2} \left[(\pi - 0) - (0 - 0)\right]$

$= \frac{\pi I_m^2}{2}$

$\therefore I = \sqrt{\frac{\text{Area of first half cycle of squared wave}}{\text{base}}}$

$= \sqrt{\frac{\pi I_m^2}{2} \times \frac{1}{\pi}}$

$= \sqrt{\frac{I_m^2}{2}}$

$= \frac{I_m}{\sqrt{2}} = 0.707 I_m$

Hence, for a sinusoidal current,

R.M.S. value of current = 0.707 x maximum value of current.

Similarly, $E = 0.707 \, E_m$

3.7 Average Value

The arithmetical average of all the values of an alternating quantity over one cycle is called **average value.**

In the case of a symmetrical wave e.g. sinusoidal current or voltage wave, the positive half is exactly equal to the negative half, so that the average value over the entire cycle is zero. Hence, in this case, the average value is obtained by adding or integrating the instantaneous values of current over one alternation (half-cycle) only.

The equation of a sinusoidally varying voltage

Is given by $e = E_m \sin\theta$.

Let us take an elementary strip of thickness $d\theta$ in the first half-cycle as shown in Fig.3.6. let the mid-ordinate of this strip be 'e'.

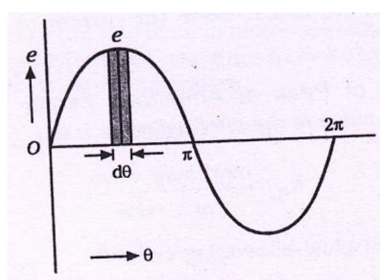


Fig. 3.6

Area of the strip = $e \, d\theta$

Area of first half-cycle

$= \int_0^{\pi} e \, d\theta$

$= \int_0^{\pi} E_m \sin\theta \, d\theta$ $\qquad (\because e = E_m \sin\theta)$

$= E_m \int_0^{\pi} \sin\theta \, d\theta$

$= E_m \left[-\cos\theta\right]_0^{\pi} = 2E_m$

$\therefore$ Average value, $E_{av} = \frac{\text{Area of half cycle}}{\text{base}} = \frac{2E_m}{\pi}$

Or $E_{av} = 0.637$ Em

In a similar manner, we can prove that, for alternating current varying sinusoidally,

$I_{av} = 0.637\ I_m$

$\therefore$ **Average value of current = 0.637 x maximum value**

3.8 Form Factor and crest or peak or Amplitude Factor (K_f)

A definite relationship exists between crest value (or peak value), average value and r.m.s. value of an alternating quantity.

1. Form Factor: The ratio of effective value (or r.m.s. value) to average value of an alternating quantity (voltage or current) is called form factor, i.e.

$$\text{From Factor, } K_f = \frac{\text{rm s value}}{\text{average value}}$$

For sinusoidal alternating current,

$$K_f = \frac{0.707 I_m}{0.637 I_m} = 1.11$$

For sinusoidal alternating voltage,

$$K_f = \frac{0.707 I_m}{0.637 I_m} = 1.11$$

Hence, the R.M.S. value (of current or voltage) is 1.11 times its average value.

2. Crest or Peak or Amplitude Factor (Ka): It is defined as the ratio of maximum value to the effective value (r.m.s. value) of an alternating quantity. i.e.,

$$Ka = \frac{\text{maximum value}}{\text{r.m.s.value}}$$

For sinusoidal alternating current,

$$Ka = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = \sqrt{2} = 1.414$$

For sinusoidal alternating voltage,

$$Ka = \frac{E_m}{\frac{E_m}{\sqrt{2}}} = 1.414$$

The knowledge of Crest Factor is particularly important in the testing of dielectric strength of insulating materials; this is because the breakdown of insulating materials depends upon the maximum value of voltage.

3.9 Phase

An alternating voltage or current changes in magnitude and direction at every instant. So, it is necessary to know the condition of the alternating quantity at a particular instant. The location of the condition of the alternating quantity at any particular instant is called its phase.

We may define the phase of an alternating quantity at any particular instant as the fractional part of a period or cycle through which the quantity has advanced from the selected origin.

Taking an example, the phase of current at point A (+ve maximum value) is T/4 second, where T is the time period, or expressed in terms of angle, it is $\pi/2$ radians (Fig.3.7).

in other words, it means that the condition

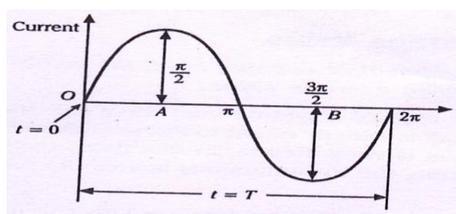


Fig. 3.7

of the wave, after having advanced through π/2 radians (900) from the selected origin (i.e.,0) is that it is maximum value (in the positive direction).similarly, -ve maximum value is reached after 3π/2 radians (2700) from the origin, and the phase of the current at point B is 3T/4 second.

3.10 Phase Difference (Lagging or Leading of Sinusoidal wave)

When two alternating quantities, say, two voltages or two currents or one voltage and one current are considered simultaneously, the frequency being the same, they may not pass through a particular point at the same instant.

One may pass through its maximum value at the instant when the other passes through a value other than its maximum one. These two quantities are said to have a phase difference. Phase difference is specified either in degrees or in radians.

The phase difference is measured by the angular difference between the points where the two curves cross the base or reference line in the same direction.

The quantity ahead in phase is said to lead the other quantity, whereas the second quantity is said to lag behind the first one. In Fig.3.8, current I_1, represented by vector 0A, leads the current I_2, represented by vector 0B, by ϕ, or current I_2 lags behind the current I_1 by ϕ.

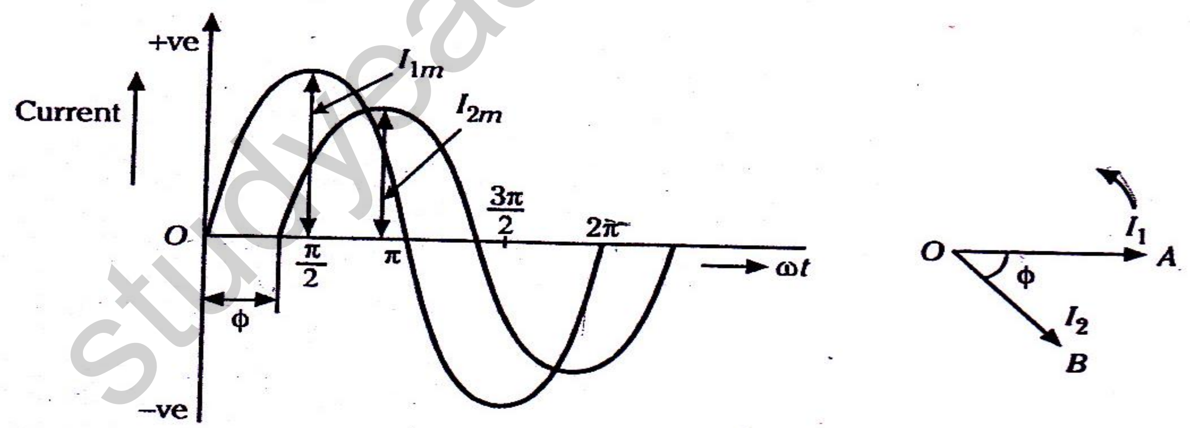


Fig. 3.8

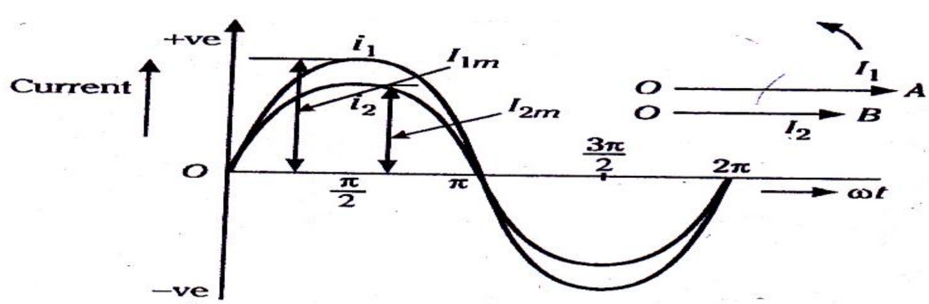


Fig. 3.9

The leading current I1 goes through its zero and maximum values first and the current I_2 goes through its zero and maximum values after time angle ϕ. The two waves representing these two currents are shown in Fig.3.8. if I_1 is taken as reference vector, two currents are expressed as

$$i_1 = I_{1m} \sin \omega t \qquad \text{and } i_2 = I_{2m} \sin (\omega t - \phi)$$

The two quantities are said to be in phase with each other if they pass through zero values at the same instant and rise in the same direction, as shown in Fig.3.9. however, if the two quantities pass through zero values at the same instant but rise in opposite, as shown in Fig.3.10, they are said to be in phase opposition i.e., the phase difference is 180^0. When the two alternating quantities have a phase difference of 90^0 or $\pi/2$ radians they are said to be in quadrature.

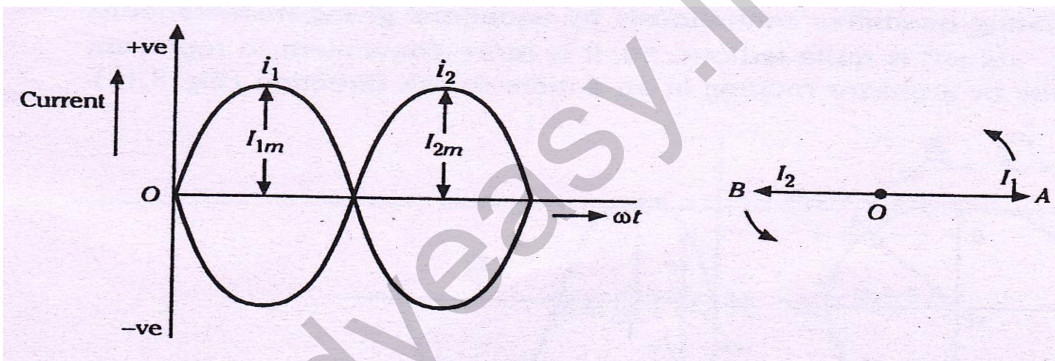


Fig. 3.10

3.11 Phasor Representation of Alternating Quantities

We know that an alternating voltage or current has sine waveform, and generators are designed to give e.m.f.s. with the sine waveforms. The method of representing alternating quantities continuously by equation giving instantaneous values (like e = Em sin ωt) is quite tedious. So, it is more convenient to represent a sinusoidal quantity by a phasor rotating in an anticlockwise direction (Fig.3.12).

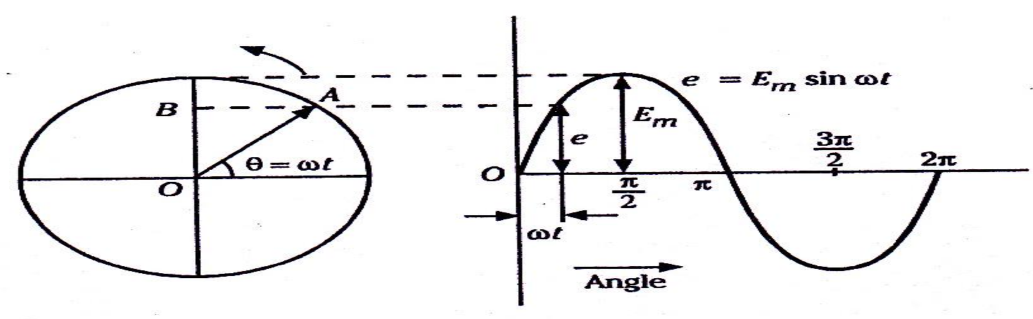


Fig. 3.12

While representing an alternating quantity by a phasor, the following points are to be kept in mind:

i) The length of the phasor should be equal to the maximum value of the alternating quantity.
ii) The phasor should be in the horizontal position at the alternating quantity is zero and is increasing in the positive direction.
iii) The inclination of the line with respect to some axis of reference gives the direction of that quantity and an arrow-head placed at one end indicates the direction in which that quantity acts.
iv) The angular velocity in an anti-clockwise direction of the phasor should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

Consider phasor 0A, which represents the maximum value of the alternating e.m.f. and its angle with the horizontal axis gives its phase (Fig.3.12). now, it will be seen that the projection of this phasor 0A on the vertical axis will give the instantaneous value of e.m.f.

$\therefore$ 0B = 0A sin ωt

Or e = 0A sin wt

= Em sin ωt

Note: The term 'phasor' is also known as 'vector'.

a) $8+j6=\sqrt{8^2+6^2} < \tan^{-1} 0.75 = 10 < 36.9^0$
b) $-10-j7.5=\sqrt{(-10)^2+(-7.5)^2} < \tan^{-1} 0.75$

$= 12.5 < \tan^{-1} 0.75$

This vector also falls in the third quadrant, so, following the same reasoning as mentioned in method 1, the angle when measured in CCW direction is

$\tan^{-1}$ 0.75)

$= 180^0+36.9^0=216.9^0$

Measured in CCW direct from + ve co-ordinate of x-axis, the angle is

- $(360^0 - 216.9^0) = -143.1^0$

So this expression is written as $12.5 < -143.1^0$

So, expression (ii) is rewritten as

$10 < 36.9^0 \times 12.5 < -143.1^0$

$125 < -106.2^0$ which is the same as before.

3.16 A.C. circuit

The path for the flow of alternating current is called on a.c. circuit.

In a d.c. circuit, the current/flowing through the circuit is given by the simple relation $I = \frac{V}{R}$. However, in an a.c. circuit, voltage and current change from instant to instant and so give rise to magnetic (inductive) and electrostatic (capacitive) effects. So, in an a.c. circuit, inductance and capacitance must be considered in addition to resistance.

We shall now deal with the following a.c. circuits:

i) AC circuit containing pure ohmic resistance only.
ii) AC circuit containing pure inductance only.
iii) AC circuit containing pure capacitance only.

3.16.1 AC circuit containing pure ohmic Resistance

When an alternating voltage is applied across a pure ohmic resistance, electrons (current) flow in one direction during the first half-cycle and in the opposite direction during the next half-cycle, thus constituting alternating current in the circuit.

Let us consider an a.c. circuit with just a pure resistance R only, as shown in Fig.3.31.

Let the applied voltage be given by the equation

$v = V_m \sin\theta = V_m \sin\omega t$ --- (i)

As a result of this alternating voltage, alternating current 'i' will flow through the circuit.

The applied voltage has to supply the drop in the resistance, i.e.,

$v = iR$

Substituting the value of 'v' from eqn.(i), we get

$V_m \sin\omega t = iR$ or $i = \frac{V_m}{R} \sin\omega t$ ---(ii)

The value of the alternating current 'i' is maximum when $\sin\omega t = 1$,

i.e., $I_m = \frac{V_m}{R}$

∴ Eqn.(ii) becomes,

$i = I_m \sin\omega t$ --- (iii)

From eqns.(i) and (ii), it is apparent that voltage and current are in phase with each other. This is also indicated by the wave and vector diagram shown in Fig. 3.32.

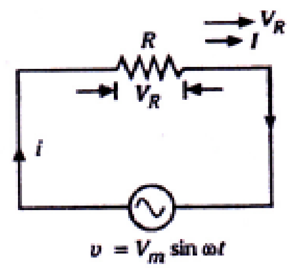


Fig. 3.31

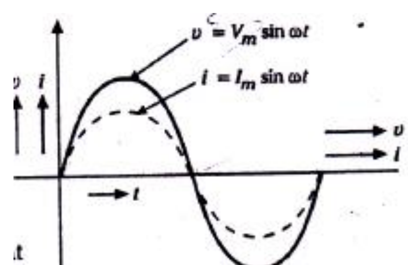


Fig. 3.32

Power: The voltage and current are changing at every instant.

∴ Instantaneous power, $P = V_m \sin\omega t . = I_m \sin\omega t$

$= V_m I_m \sin^2\omega t$

$= V_m I_m \frac{(1-\cos 2\omega t)}{2}$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2}.\cos 2\omega t$$

Thus instantaneous power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2}\cos 2\omega t$ of frequency double that of voltage and current waves.

The average value of $\frac{V_m I_m}{2}\cos 2\omega t$ over a complete cycle is zero.
So, power for the complete cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

or $P = V1$ watts

Where V = r.m.s. value of applied voltage
I = r.m.s. value of the current

Power curve

The power curve for a purely resistive circuit is shown in Fig. 3.33. It is apparent that power in such a circuit is zero only at the instants a,b and c, when both voltage and current are zero, but is positive at all other instants. in other words, power is never negative, so that power is always lost in a resistive a.c. circuit. This power is dissipated as heat.

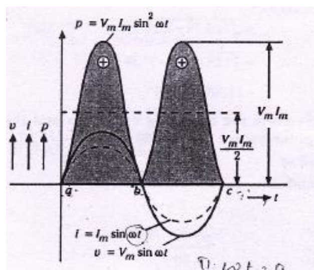


Fig. 3.33

3.16.2 A.C. circuit containing pure Inductance

An inductive coil is a coil with or without an iron core and has negligible resistance. In practice, pure inductance can never be had as the inductive coil has always a small resistance. However, a coil of thick copper wire wound on a laminated iron core has negligible resistance, so, for the purpose of our study, we will consider a purely inductive coil.

On the application of an alternating voltage (Fig.3.34) to a circuit containing a pure inductance, a back e.m.f. is produced due to the self-inductance of the coil. This back e.m.f. opposes the rise or fall o f current, at every stage. Because of the absence of voltage drop, the applied voltage has to overcome this self-induced e.m.f. only.

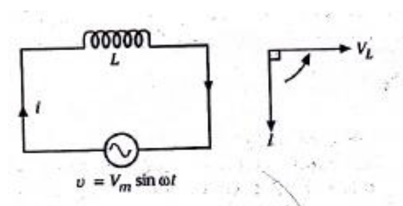


Fig. 3.34

Let the applied voltage be $v = V_m \sin \omega t$, and the self-inductance of the coil = L henry.

Self-induced e.m.f. in the coil, $e_L = -L\frac{di}{dt}$

Since applied voltage at every instant is equal and opposite to the self-induced e.m.f., i.e. $v = -e_L$

$$\therefore \quad V_m \sin \omega t = -\left(-L\frac{di}{dt}\right)$$

or $\quad di = \frac{V_m}{L} \sin \omega t \; dt$

Integrating both sides, we get

$$i = \frac{V_m}{L} \int \sin \omega t \; dt$$

or $\quad i = \frac{V_m}{\omega L}(-\cos \omega t) + A$

Where A is a constant of integration which is found to be zero from initial conditions.

So, $\quad i = \frac{-V_m}{\omega L} \cos \omega t$

Or $\quad i = \frac{V_m}{\omega L} \sin \omega t$

Current will be maximum when $\sin \omega t = 1$, hence, the value of maximum current, $I_m = \frac{V_m}{\omega L}$, and instantaneous current may be expressed as $i = I_m \sin \omega t$.

From the expressions of instantaneous applied voltage ($v = V_m \sin \omega t$) and the instantaneous current flowing through a purely inductive coil, it is clear that the current lags behind the voltage by $\frac{\pi}{2}$ as shown in Fig. 3.35.

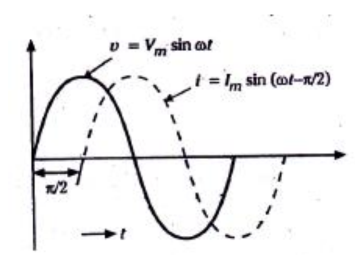


Fig. 3.35

Inductive Reactance:

ωL in the expression $I_m = \frac{V_m}{\omega L}$ is known as inductive reactance and is denoted by X_L, i.e., $X_L = \omega L$. If 'L' is in henry and 'ω' is in radians per second, then X_L will be in ohms. So, inductive reactance plays the part the part of resistance.

Power: Instantaneous Power,

$$P = v \times i = V_m \sin \omega t \,.\, I_m \sin \omega t$$

$$= - V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{-V_m I_m}{2} \sin 2\omega t$$

The power measured by a wattmeter is the average value of 'p', which is zero since average of a sinusoidal quantity of double frequency over a complete cycle is zero. Put in mathematical terms,

Power for the whole cycle, $P = - \frac{-V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, dt = 0$

Hence, power absorbed in a pure inductive circuit is zero.

Power curve

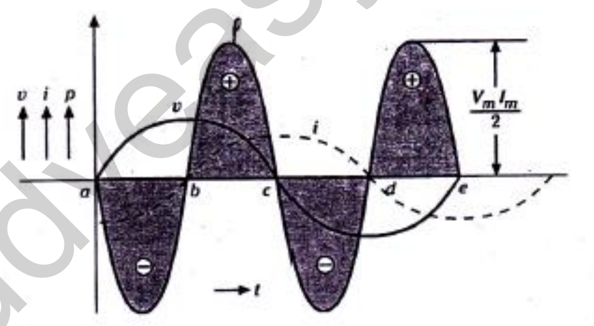


Fig. 3.36

The power curve for a pure inductive circuit is shown in Fig. 3.36. This indicates that power absorbed in the circuit is zero. At the instants a,c and e, voltage is zero, so that power is zero: it is also zero at points b and d when the current is zero. Between a and b voltage and current are in opposite directions, so that power is negative and energy is taken from the circuit. Between b and c voltage and current are in the same direction, so that power is positive and is put back into the circuit. Similarly, between c and d, power is taken from the circuit and between d and e it is put into the circuit. Hence, net power is zero.

3.16.3 AC circuit containing pure capacitance

When an alternating voltage is applied across the plates of a capacitor, the capacitor is charged in one direction and then in the opposite direction as the voltage reverses. With

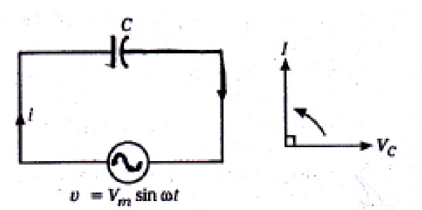


Fig. 3.38

reference to Fig. 3.38,

Let alternating voltage represented by $v = V_m \sin \omega t$ be applied across a capacitor of capacitance
C Farads.

Instantaneous charge, $q = cv = CV_m \sin \omega t$

Capacitor current is equal to the rate of change of charge,

or

$$\frac{dq}{dt} = \frac{d}{dt}(CV_m \sin \omega t)$$

$$= \omega CV_m \cos \omega t$$

or $$i = \frac{V_m}{\frac{1}{\omega c}} \sin \widehat{\omega t}$$

The current is maximum when t = 0

$$\therefore I_m = \frac{V_m}{\frac{1}{\omega c}}$$

Substituting $\frac{V_m}{\frac{1}{\omega c}} = I_m$ in the above expression for instantaneous current, we get

$$i = I_m \sin \widehat{\omega t}$$

Capacitive Reactance: $\frac{1}{\omega c}$ in the expression $I_m = \frac{V_m}{\frac{1}{\omega c}}$ is known as capacitive reactance and is denoted by X_c.

i.e., $X_c = \frac{1}{\omega c}$

If C is farads and 'ω' is in radians, then X_c will be in ohms.

It is seen that if the applied voltage is given by $v = V_m \sin \omega t$, then the current is given by $i = I_m \sin \widehat{\omega t}$ this shows that the current in a pure capacitor leads its voltage by a quarter cycle as shown in Fig. 3.39, or phase difference between its voltage and current is $\frac{\pi}{2}$ with the current leading.

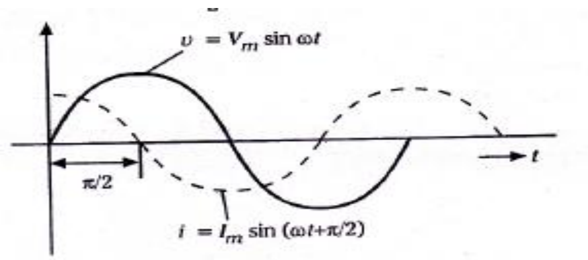


Fig. 3.39

Power: Instantaneous Power,

$$P = vi$$

$$= V_m \sin \omega t . I_m \sin \omega t$$

$$= V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t$$

Power for the complete cycle

$$= \frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t \; dt = 0$$

Hence power absorbed in a capacitive circuit is zero.

Power curves (Fig. 3.40)

At the instants b,d, the current is zero, so that power is zero; it is also zero at the instants a,c and e, when the voltage is zero. Between a and b, voltage and current are in the same direction, so that power is positive and is being put back in the circuit. Between b and c, voltage and current are in the opposite directions, so that power is negative and energy is taken from the circuit. Similarly, between c and d, power is put back into the circuit, and between d and e it is taken from the circuit.

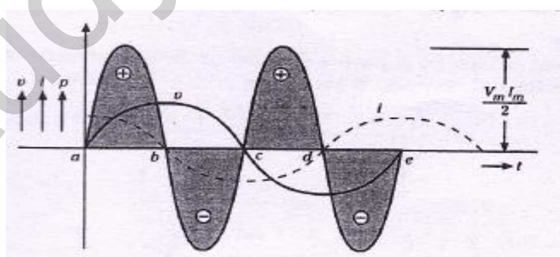


Fig. 3.40

Therefore, power absorbed in a pure capacitive circuit is zero.

3.17 Series R-L circuit

Let us consider an a.c. circuit containing a pure resistance R ohms and a pure inductance of L henrys, as shown in Fig. 3.43.

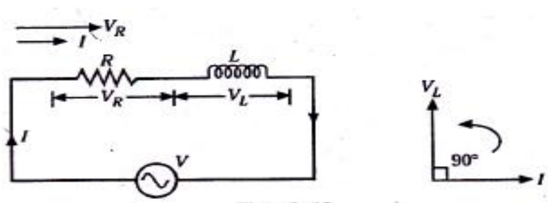


Fig. 3.43

Let V = r.m.s. value of the applied voltage

I = r.m.s. value of the current

Voltage drop across R, $V_R = IR$ (in phase with I)

Voltage drop across L, $V_L = IX_L$ (leading I by 90^0)

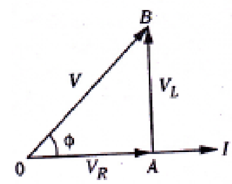


Fig. 3.44

The voltage drops across these two circuit components are shown in Fig. 3.44, where vector OA indicates V_R and AB indicates V_L. The applied voltage V is the vector sum of the two, i.e., OB.

$$\therefore \quad V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$

$$\therefore \quad I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

The term $\sqrt{R^2 + X_L^2}$ offers opposition to current flow and is called the impedance (Z) of the circuit. It is measured in ohms.

$$\therefore \quad I = \frac{V}{Z}$$

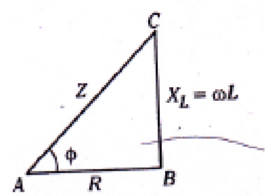


Fig. 3.45

Referring to the impedance triangle ABC, (Fig. 3.45)

$$Z^2 = R^2 + X_L^2$$

or $(\text{impedance})^2 + (\text{reactance})^2$

Referring back to Fig. 3.44, we observe that the applied voltage V leads the current I by an angle ϕ.

$$\tan\phi = \frac{V_L}{V_R} = \frac{I.X_L}{I.R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{reactance}}{\text{resistance}}$$

$$\therefore \quad \phi = \tan^{-1}\frac{X_L}{R}$$

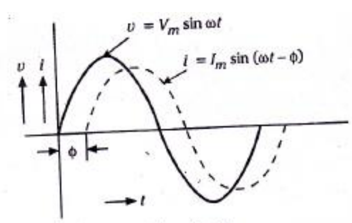


Fig. 3.46

The same feature is shown by means of waveforms (Fig. 3.46). We observe that circuit current lags behind applied voltage by an angle ϕ.

So, if applied voltage is expressed as $v = V_m \sin \omega t$, the current is given by $i = I_m(\sin \omega t - \phi)$, Where $I_m = \frac{V_m}{Z}$.

Definition of Real power, Reactive Power, Apparent power and power Factor

Let a series R-L circuit draw a current I (r.m.s. value) when an alternating voltage of r.m.s. value V is applied to it. Suppose the current lags behind the applied voltage by an angle ϕ as shown in Fig. 3.47.

Power Factor and its signifies

Power Factor may be defined as the cosine of the angle of lead or lag. In Fig. 3.47, the angle of lag is shown. Thus power Factor = cos ϕ.

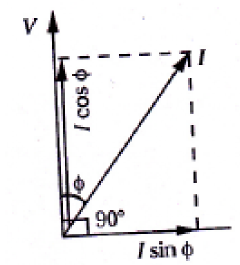


Fig. 3.47

In addition to having a numerical value, the power factor of a circuit carries a notation that signifies the nature of the circuit, i.e., whether the equivalent circuit is resistive, inductive or capacitive. Thus, the p.f. might be expressed as 0.8 lagging. The lagging and leading refers to the phase of the current vector with respect to the voltage vector. Thus, a lagging power factor means that the current lags the voltage and the circuit is inductive in nature. However, in the case of leading power factor, the current leads the voltage and the circuit is capacitive.

Apparent Power: The product of r.m.s. values of current and voltage, VI, is called the apparent power and is measured in volt-amperes (VA) or in kilo-volt amperes (KVA).

Real Power: The real power in an a.c. circuit is obtained by multiplying the apparent power by the factor and is expressed in watts or killo-watts (kW).

Real power (W) = volt-amperes (VA) × power factor cos ϕ

or **Watts = VA cos ϕ**

Here, it should be noted that power consumed is due to ohmic resistance only as a pure inductance does not consume any power.

Thus, $P = V I \cos\phi$

$$\cos\phi = \frac{R}{Z} \text{ (refer to the impedance triangle of Fig. 3.45)}$$

$$\therefore \quad P = V I \times \left[\frac{R}{Z}\right]$$

$$= \left[\frac{V}{Z}\right] \times IR = I^2R$$

or **$P = I^2R$ watts**

Reactive Power: It is the power developed in the inductive reactance of the circuit. The quantity VI sin ϕ is called the reactive power; it is measured in reactive volt-amperes or vars (VAr).

The power consumed can be represented by means of waveform in Fig. 3.48.

We will now calculate power in terms of instantaneous values.

Instantaneous power, $P = vi = V_m \sin \omega t \times I_m \sin(\omega t - \phi)$

$$= V_m I_m \sin \omega t \sin(\omega t - \phi)$$

$$= \frac{1}{2} V_m I_m [\cos \phi - \cos(2\omega t - \phi)]$$

This power consists of two parts:

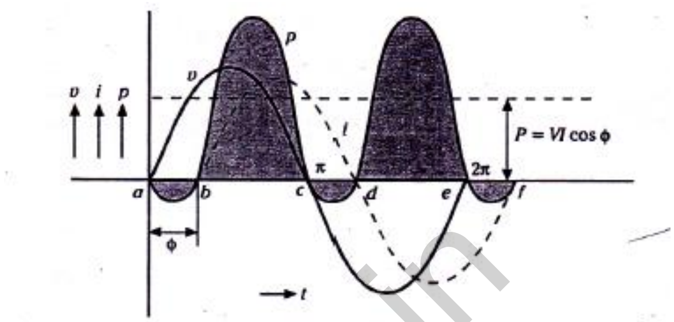


Fig. 3.48

i) Constant part $\frac{1}{2} V_m I_m \cos \phi$ which contributes to real power.

ii) Sinusoidally varying part $\frac{1}{2} V_m I_m \cos(2\omega t - \phi)$, whose frequency is twice that of the voltage and the current, and whose average value over a complete cycle is zero (so it does not contribute to any power).

So, average power consumed, $P = \frac{1}{2} V_m I_m \cos \phi$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \phi$$

$$\mathbf{= V\,I\,\cos \phi}$$

Where V and I are r.m.s. values

Power curves: The power curve for R-L series circuit is shown in Fig. 3.48. The curve indicates that the greater part is positive and the smaller part is negative, so that the net power over the cycle is positive.

During the time interval a to b, applied voltage and current are in opposite directions, so that power is negative. Under such conditions, the inductance L returns power to the circuit. During the period b to c, the applied voltage and current are in the same direction so that power is positive , and therefore, power is put into the circuit. In a similar way, during the period c to d, inductance L returns power to the circuit while between d and e, power is put into the circuit. The power absorbed by resistance R is converted into heat and not returned.

3.18 Series R – C circuit

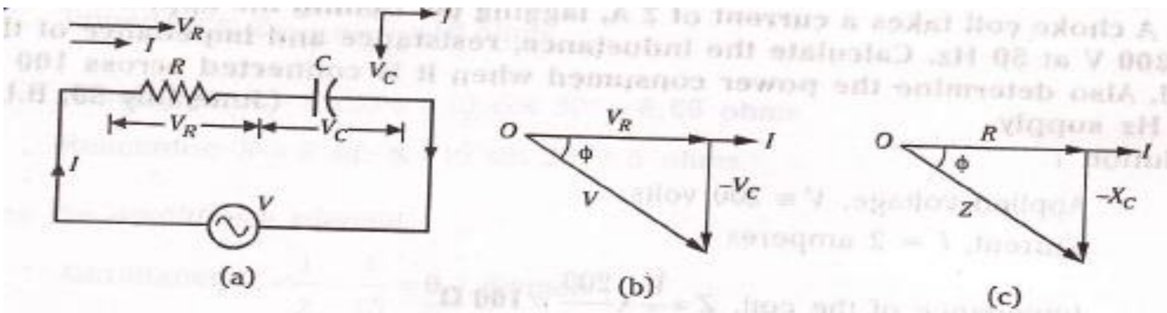


Fig. 3.52

Consider an a.c. circuit containing resistance R ohms and capacitance C farads, as shown in the fig. 3.52(a).

Let V = r.m.s. value of voltage

I = r.m.s. value of current

∴ voltage drop across R, $V_R = IR$ - in phase with I

Voltage drop across C, $V_C = IX_C$ - lagging I by $\frac{\pi}{2}$

The capacitive resistance is negative, so V_C is in the negative direction of Y – axis, as shown in the fig. 3.52(b).

We have $V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (-IX_C)^2}$

$= \sqrt{R^2 + X_C{}^2}$

Or $I = \frac{V}{\sqrt{R^2+X_C{}^2}} = \frac{V}{Z}$

The denominator, Z is the impedance of the circuit, i.e., $Z = \sqrt{R^2 + X_C{}^2}$. fig. 3.52(c) depicts the impedance triangle.

Power factor, $\cos\phi = \frac{R}{Z}$

Fig. 3.52(b) shows that I leads V by an angle ϕ, so that $\tan\phi = \frac{-X_C}{R}$

This implies that if the alternating voltage is $v = V_m \sin\omega t$, the resultant current in the R – C circuit is given by $i = I_m \sin(\omega t + \phi)$, such that current leads the applied voltage by the angle ϕ. The waveforms of fig. 3.53 depict this

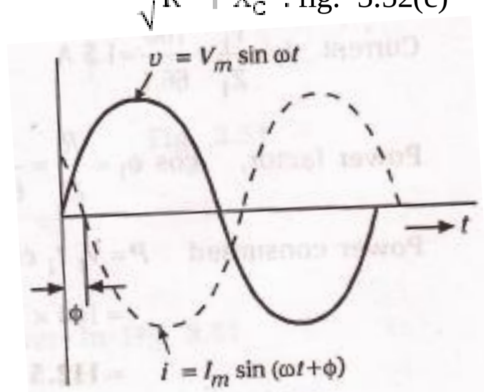


Fig. 3.53

Power: Average power, $P = v \times I = VI \cos\phi$ (as in sec. 3.17).

Power curves: The power curve for R – C series circuit is shown in fig. 3.54. The curve indicates that the greater part is positive and the smaller part is negative, so that the net power is positive.

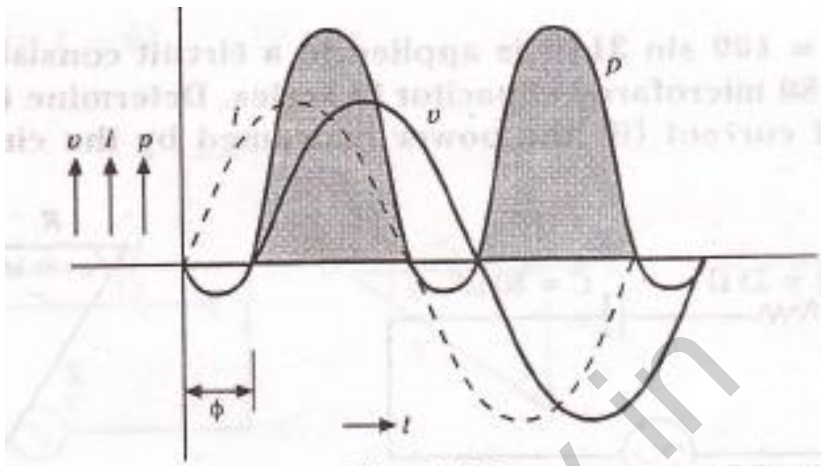


Fig. 3.54

3.19 Resistance, Inductance and capacitance in series

Consider an a.c. series circuit containing resistance R ohms, Inductance L henries and capacitance C farads, as shown in the fig. 3.59.

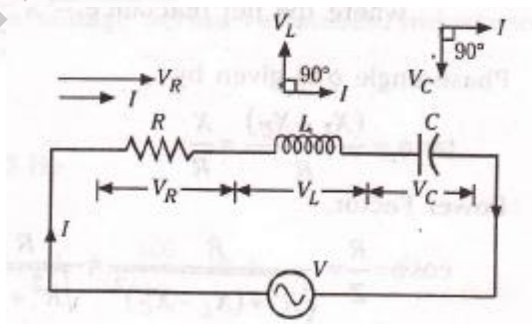


Fig. 3.59

Let V = r.m.s. value of applied voltage
I = r.m.s. value of current

∴ voltage drop across R, $V_R = IR$ - in phase v
voltage drop across L, $V_L = I.X_L$ - lagging I by 90^0
Voltage drop across C, $V_C = IX_C$ - lagging I by 90^0

Referring to the voltage triangle of Fig. 3.60, OA represents V_R, AB and AC represent inductive and capacitive drops respectively. We observe that V_L and V_C are 180^0 out of phase.

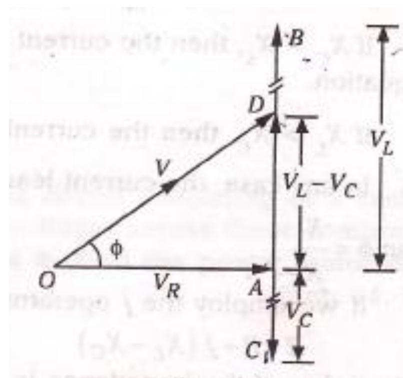


Fig. 3.60

Thus, the net reactive drop across the combination is

$$AD = AB - AC$$
$$= AB - BD \quad (\because BD = AC)$$
$$= V_L - V_C$$
$$= I(X_L - X_C)$$

OD, which represents the applied voltage V, is the vector sum of OA and AD.

$$\therefore \quad OD = \sqrt{OA^2 + AD^2} \qquad OR \quad V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$
$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$Or \quad I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + X^2}} = \frac{V}{Z}$$

The denominator $\sqrt{R^2 + (X_L - X_C)^2}$ is the impendence of the circuit.

So (impedance)^2 = (resistance)^2 + (net reactance)^2

Or $Z^2 = R^2 + (X_L - X_C)^2 = R^2 + X^2$

Where the net reactance = X (fig. 3.61)

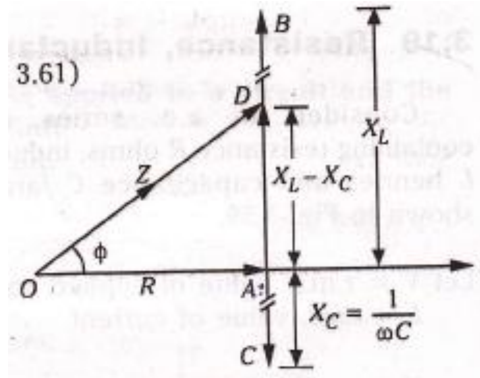


Fig. 3.61

Phase angle ϕ is given by

$$\tan\phi = \frac{(X_L - X_C)}{R} = \frac{X}{R}$$

power factor,

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + X^2}}$$

Power = VI cos ϕ

If applied voltage is represented by the equation $v = V_m \sin\omega t$, then the resulting current in an R – L – C circuit is given by the equation

$$i = I_m \sin(\omega t \pm \phi)$$

If $X_C > X_L$, then the current leads and the +ve sign is to be used in the above equation.

If $X_L > X_C$, then the current lags and the –ve sign is to be used.

If any case, the current leads or lags the supply voltage by an angle ϕ, so that $\tan\phi = \frac{X}{R}$.

If we employ the j operator (fig. 3.62), then we have

$$Z = R + j(X_L - X_C)$$

The value of the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The phase angle $\phi = \tan^{-1}\frac{(X_L - X_C)}{R}$

$$Z\angle\phi = Z\angle \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$$

$$= Z\angle \tan^{-1}\left[\frac{X}{R}\right]$$

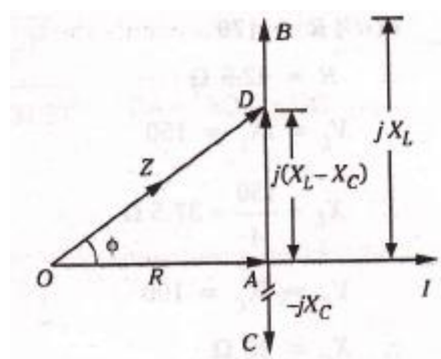


Fig. 3.62

3.20 Parallel AC circuits

In a parallel a.c. circuit, the voltage across each branch of the circuit is the same whereas current in each branch depends upon the branch impedance. Since alternating currents are vector quantities, total line current is the vector sum of branch currents.

The following are the three methods of solving parallel a.c. circuits:

a) Vector method.
b) Admittance method.
c) Symbolic or j- method.

3.20.1 Vector method

In this method the total line current is found by drawing the vector diagram of the circuit. As voltage is common, it is taken as the reference vector and the various branch currents are represented vectorially. The total line current can be determined from the vector diagram either by the parallelogram method or by the method of components.

Branch 1

Impedance $Z_1 = \sqrt{R_1^2 + X_L^2}$

Current $I_1 = \frac{V}{Z_1}$

$\cos\phi_1 = \frac{R_1}{Z_1}$ or $\phi_1 = \cos^{-1}\left(\frac{R_1}{Z_1}\right)$

Current I_1 lags behind the applied voltage by ϕ (fig. 3.65).

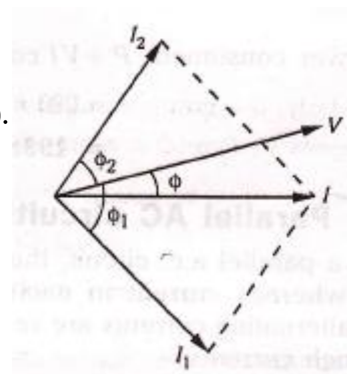


Fig. 3.65

Branch 2

Impedance $Z_2 = \sqrt{R_2^2 + X_C^2}$

Current $I_2 = \frac{V}{Z_2}$

$\cos\phi = \frac{R_2}{Z_2}$ $\phi_2 = \cos^{-1}\left(\frac{R_2}{Z_2}\right)$

Current I_2 leads V by ϕ_2 (fig. 3.65).

Resultant current : The total line current I is the vector sum of the branch currents I_1 and I_2 and is found by using the parallelogram law of vectors, as shown in fig. 3.65.

The second method is the method of components i.e., resolving the branch currents I_1 and I_2 along the x- axis and y- axis and then finding the resultant of these components (fig. 3.66).

Let the resultant current be I and ϕ be its phase angle, as shown in fig. 3.66 (b). Then the components of I along X- axis is equal to the algebraic sum of the components of branch currents I_1 and I_2 along the X- axis (active components).

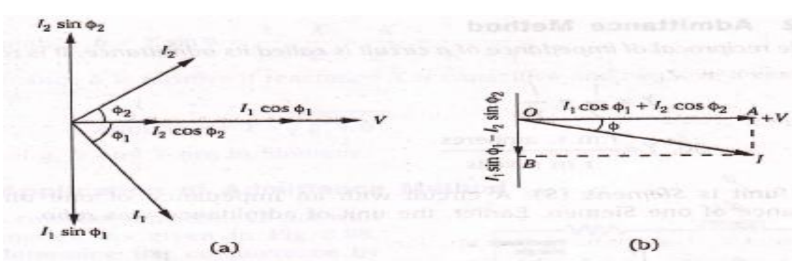


Fig. 3.66

Similarly, the component of I along Y- axis is equal to the algebraic sum of the components of I_1 and I_2 along Y- axis i.e,

Component of resultant current along Y- axis

= algebraic sum of I_1 and I_2 along X – axis

or $I\cos\phi = I_1\cos\phi_1 + I_2\cos\phi_2$

Component of resultant current along Y – axis

$$\therefore \quad I = \sqrt{(I\cos\phi)^2 + (I\sin\phi)^2}$$

$$= \sqrt{(I_1\cos\phi_1 + I_2\cos\phi_2)^2 + (I_1\sin\phi_1 - I_2\sin\phi_2)^2}$$

and $\tan\phi = \frac{I_1\sin\phi_1 - I_2\sin\phi_2}{I_1\cos\phi_1 + I_2\cos\phi_2}$

If $\tan\phi$ is positive, current leads and if $\tan\phi$ is negative, then the current lags behind applied voltage V. power factor for the entire circuit

$$\cos\phi = \frac{I_1\cos\phi_1 + I_2\cos\phi_2}{I}$$

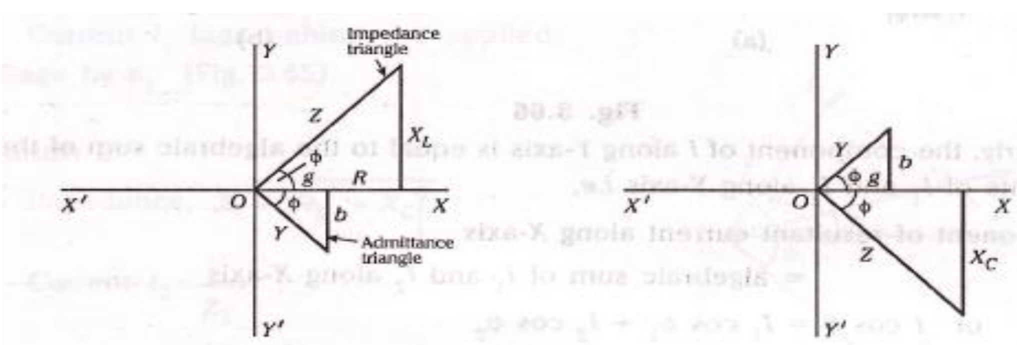
3.20.2 Admittance Method

The reciprocal of impedance of a circuit is called its admittance. It is represented by Y.

$$Y = \frac{1}{Z} = \frac{I}{V}$$

So, $Y = \frac{\text{r.m.s.amperes}}{\text{r.m.s volts}}$

Its unit is Siemens (S). A circuit with an impedance of one ohm has an admittance of one siemen. Earlier, the unit of admittance was mho.



(a) Inductive Circuit (b) Capacitive circuit

Fig. 3.67

Just as impedance Z of a circuit had two rectangular components, resistance R and reactance X, admittance Y also has two rectangular components known as conductance **g** and susceptance **b**. fig. 3.67 shows the impendence triangle and the admittance triangle. It is clear the admittance has two components **g** and **b.** The component **g** along the X- axis is the conductance which is the reciprocal of resistance. The component **b** is called susceptance, which is the reciprocal of reactance.

In fig. 3.67(a), the impedance and admittance triangles for an inductive circuit are shown. It is apparent that susceptance **b** is negative, being below X – axis. Hence inductive susceptance is negative. In fig. 3.67 (b), the impedance and admittance triangles for capacitive circuit is shown. It is evident that susceptance is positive, being above the X – axis; hence, capacitive susceptance is positive.

Relations

Conductance **g** = Y cos ϕ

Or $g = \frac{1}{Z} \cdot \frac{R}{Z} = \frac{R}{Z^2} = \frac{R}{R^2 + X^2}$

Conductance is always positive.

Susceptance **b** = Y sin $\phi = \frac{1}{Z} \cdot \frac{X}{Z} = \frac{X}{Z^2} = \frac{X}{R^2 + X^2}$

Susceptance **b** is positive if reactance **X** is capacitive and negative if reactance is inductive.

Admittance $Y = \sqrt{g^2 + b^2}$

The units of g, b and y are in Siemens.

3.20.3 Application of admittance method

Let us consider a parallel circuit with three branches, as given in fig. 3.68. we can determine the conductors by just adding the conductances of the three branches. In a like manner, susceptance is determined by the algebraic addition of the susceptances of the different branches.

Total conductance,

$G = g_1 + g_2 + g_3$

Total susceptance

$B = (-b_1) + (-b_2) + b_3$

$\therefore$ Total admittance $Y = \sqrt{G^2 + B^2}$

Total current $I = VY$

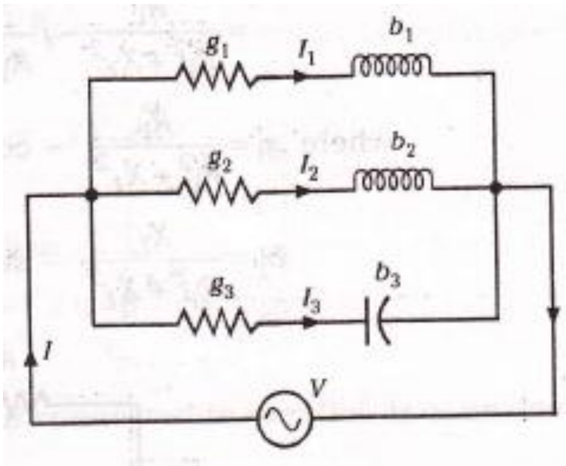


Fig. 3.68

Power factor, $\cos\phi = \frac{G}{Y}$

3.20.4 Symbolic or j- method

Let us take the parallel two – branch circuit of fig. 3.69, with the same p.d. across the two impedances Z_1 and Z_2 .

$I_1 = \frac{V}{Z_1}$ and $I_2 = \frac{V}{Z_2}$

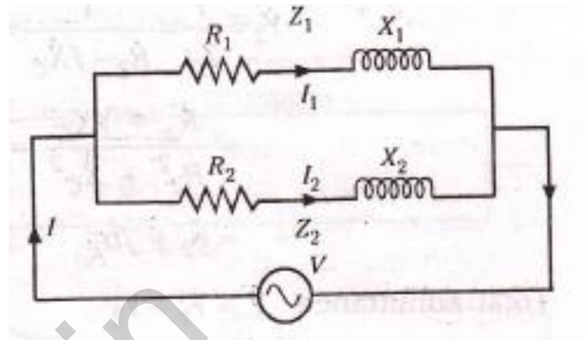


Fig. 3.69

$$\text{Total current } I = I_1 + I_2 = \frac{V}{Z_1} + \frac{V}{Z_2} = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$$
$$= V(Y_1 + Y_2)$$
$$= VY$$

Where the total admittance $Y = Y_1 + Y_2$

We should note that admittances are added for parallel branches, whereas impedances are added for series branches. Both admittances and impedances are complex quantities, so all additions have to be performed in complex form.

In case of the two parallel branches of fig. 3.70,

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L} = \frac{(R_1 - jX_L)}{(R_1 + jX_L)(R_1 - jX_L)} = \frac{R_1 - jX_L}{R_1^2 + X_L^2}$$

$$= \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1^2 + X_L^2} = g_1 - jb_1$$

Where $g_1 = \frac{R_1}{R_1^2 + X_L^2}$ ---- conductance of top branch

$b_1 = \frac{X_L}{R_1^2 + X_L^2}$ ----- susceptance of top branch

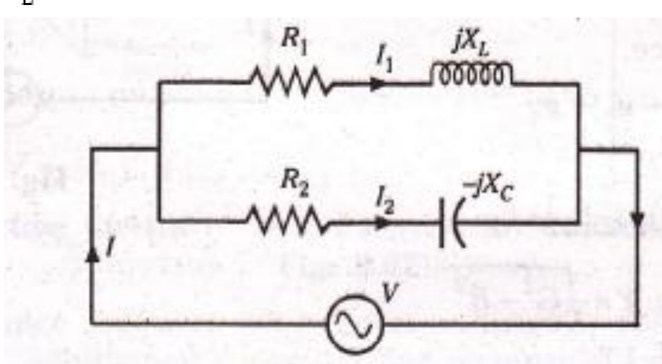


Fig. 3.70

In similar manner,

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_L} = \frac{(R_2 + jX_L)}{(R_2 - jX_C)(R_2 + jX_C)} = \frac{R_2 + jX_C}{R_2^2 + X_C^2}$$

$$= \frac{R_2}{R_2^2+X_C^2} + j\frac{X_L}{R_2^2+X_C^2} = g_2 + jb_2$$

Total admittance $Y = Y_1 + Y_2$

$$= (g_1 - jb_1) + (g_2 + jb_2)$$

$$= (g_1 + g_2) - j(b_1 - b_2)$$

$$= G - JB$$

$$Y = \sqrt{(g_1 + g_2)^2 + (b_1 - b_2)^2}$$

$$\phi = \tan^{-1}\left[\frac{b1 - b2}{g_1 + g_2}\right]$$

In polar form, admittance $Y = Y\angle\phi^0$

$$Y = \sqrt{G^2 + B^2} \angle \tan^{-1}\left(\frac{B}{G}\right)$$

Total current $I = VY$; $I_1 = VY_1$ and I_2 VY_2

$V = V\angle 0^0$ and $Y = Y\angle\phi$

So $I = VY = V\angle 0^0 \times Y\angle\phi = VY\angle\phi$

Taking a general case,

$V = V\angle\propto$ and $Y = Y\angle\beta$, then

So $I = VY = V\angle\propto \times Y\angle\beta = VY \angle \propto + \beta$

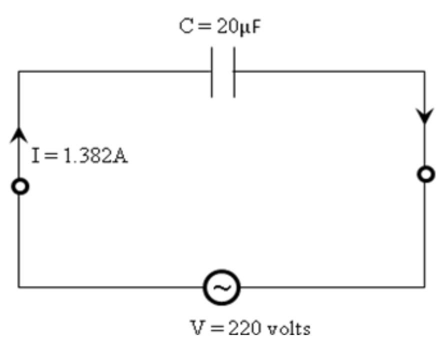
current drawn by a pure capacitor of 20 μF is 1.382 A from 220V AC supply. What is the supply frequency ?

sol. :

Given

$C = 20$ μF

$V = 220$volts



Now, $|Z| = V/I = 220/1.382 = 1.59.18$ (

Since it is a pure capacitor, its resistance is zero

$Z = -jX_c$ ∴ $|Z| = |X_c| = 159.18$ (

For capacitor, $X_c = \frac{1}{2\pi f C}$

$$\therefore \quad f = \frac{1}{2\pi X_c C} = \frac{1}{2\pi \times 159.18 \times 20 \times 10^{-6}}$$

$$\therefore \quad f = \frac{1}{0.0200} = 50Hz$$

$\therefore \qquad f = 50\text{Hz}$

An EMF whose instantaneous value is 100 sin (314t – π/4) volts is applied to a circuit and the . current flowing through it is 20sin(314t-15708) amperes. Find the frequency and the values of . circuit elements, assuming a series combination of circuit elements.

Sol. : V= 100 sin(314t – π/4) volts

I= 20sin (314t-15708) A

V_m= 100volts, $V_{rms} = \frac{I_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71$ volts

$I_m = 20A, \qquad I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.1421$ A

Comparing V=100 sin (314t – π/4) with V= $V_m \sin(\omega t + \emptyset)$

We have

ω= 314 $\qquad \therefore$ 2πf= 314

$$f = \frac{314}{2\pi} \approx 50\text{Hz}$$

$$V = 70.71\angle -45^0 \text{ volts}$$

I = 14.1421 ∠−90^0 A as 1.5708 rad≡ 90^0

Angle between V and I = 90^0 −45^0

∴ ∅ = 45^0, cos ∅ = 0.7071

Now, $Z = \frac{V}{I} = \frac{70.71}{14.1421} = 5$

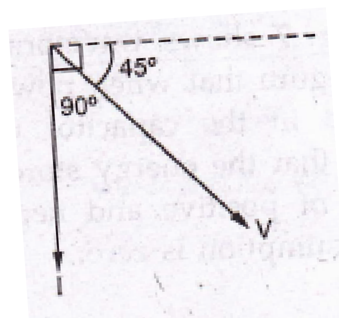


Fig. 8

As the current is lagging behind the voltage the circuit is inductive consisting of resistance and inductance reactance.

R= Z cos ∅= 5× cos 45^0 = 3.5355Ω

X_L = Z sin ∅ = 5× sin45^0 = 3.5355Ω

$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{3.5355}{2\pi \times 50} = 11\text{mH}$$

c) An inductive coil draws a current of 2A, when connected to a 230V, 50 Hz supply. The power taken by the coil is 100 watts. Calculate the resistance and inductance of the coil.

Sol. : Let r be resistance of coil while L Be inductance of coil.

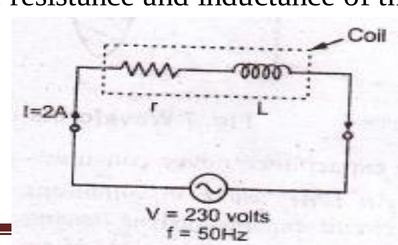


Fig. 9

$|Z| = V/I = 230/2 = 115($

Power taken by coil is consumed in resistive part of coil as inductance never consumes any part.

$P = I^2 \ r \qquad \therefore r = P/I^{\uparrow}2 = 100/(2)^{\uparrow}2 = 25($

$\therefore$ r = 25 (

Now $\qquad Z = R + jX_L$

$$|Z| = \sqrt{R^2 + X_L^2} \qquad \therefore Z^2 = R^2 + X_L^2$$

$$X_L = \sqrt{(Z^{\uparrow}2 - R^{\uparrow}2)} = \sqrt{((115)^{\uparrow}2 - (25)^{\uparrow}2)} = 112.24($$

But, $\qquad X_L = 2\pi fL$

$$\therefore \qquad L = \frac{X_L}{2\pi f} = 112.\frac{24}{2\pi \times 50} = 0.3573H$$

$\therefore \qquad$ L = 0.3573 H

A single phase voltage of (200+jo) v at 50 Hz is applied to a circuit comprising of a resistance of 20Ω inductance of 20 mH and a capacitance of 150 µF connected in series. Find,

i) Impedance of the circuit
ii) Current drawn from supply
iii) Power factor
iv) Power drawn
v) Energy stored in inductor and capacitor

and draw voltage diagram**Ans.:** Given:

R = 20Ω, L = 20 mH, C = 150µF

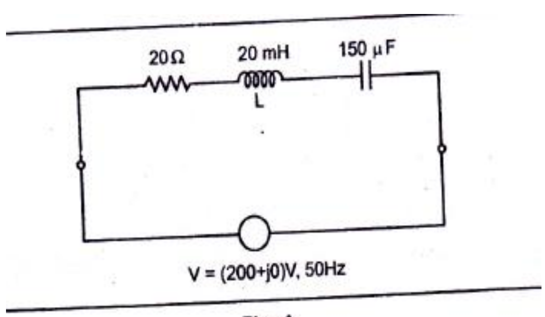


Fig. 4

i) Impedance of the circuit:

$X_L = 2\pi fL$
$= 2\pi \times 50 \times 20 \times 10^{-3}$
$= 6.28\Omega$

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}}$$

$= 21.22\Omega$

$Z = R + j\,(X_L - X_C)$

$Z = 20 + j\,(6.28 - 21.22)$

$Z = 20 - j\,14.94\ \Omega$

$\mathbf{Z = 24.96 < -36.75^0 \Omega}$

ii) Current drawn from supply:

$$|Z| = \left|\frac{V}{I}\right|$$

$$|I| = \left|\frac{V}{Z}\right| = \frac{200 < 0}{24.96 < 36.75}$$

$= 8.01 < 36.75^0\ A$

$\therefore \quad \mathbf{|I| = 8.01\ A}$

iii) Power factor:
Let current I be the reference phasor

$I = 8.01 < 0^0\ A$

$V_R = I.R. = 8.01\,(20)$

$= 160.2 < 0^0$ volts

$V_L = I\,(jX_L)$

$= (8.01 < 0^0)\,(j\,6.28)$

$= (8.01 < 0^0)\,(6.28 < 90^0)$

$= 50.30 < 90^0$ volts

$V_C = (8.01 < 0^0)\,(-j\,21.22)$

$= (8.01 < 0^0)\,(21.22 < -90^0)$

$= 169.97 < -90^0$ volts

As the circuit is capacitive current leads voltage by an angle of 36.75^0

p.f. = cos 36.75 = 0.801 lead

iv) Power drawn:

Power = VI cosØ

= (200) (8.01) (0.801)

= 1283.20 W

A circuit having a resistance of 12Ω, an inductance of 0.15 H and a capacitance of 100μF in series is connected across a 100V, 50 Hz supply. Calculate the impedance, current, the phase difference between the current and supply voltage.

Sol:

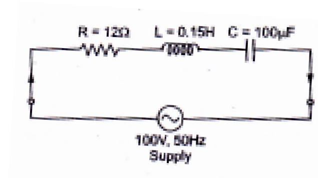


Fig. 7

$X_L = 2\pi FL = 2\pi \times 50 \times 0.15$

$= 100 \times \pi \times 0.15$

$\therefore X_L = 47.12\Omega$

$X_C = \frac{1}{2\pi fL} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$

$\therefore X_C = 31.83\Omega$

$Z = R + jX_L - jX_C = 12 + j47.12 - j31.83$

$= 12 + j15.29\Omega$

$V = 100\text{volts} = 100\angle 0^0$

$Z = 12 + j15.29 = 19.43\ \angle 51.87^0\Omega$

$I = \frac{V}{Z} = \frac{100\ (0^0}{19.43\ (51.87^0} = 5.146\angle 51.87^0\Omega A$

$\therefore \quad I = 5.146A$

p.f. $= \cos\phi = \cos 51.87^0 = 0.6174$ lag, $\quad \phi = 51.87^0$

c) Two circuits with impedances of Z1 = 10 + j15Ω and Z2 = 6-j8Ω are connected in parallel. If the supply current is 20A, what is the power dissipated in each branch?

Sol :

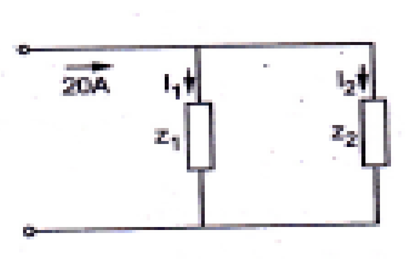


Fig. 8

$Z_1 = 10 + j15\Omega = 18.027\ \angle\ 56.30^0\Omega$

$Z_2 = 6\text{-}j8\Omega = 10\ \angle 53.13^0\Omega$

$I = 20A$

$I_1 = \frac{I \times Z_2}{Z_1 + Z_2} = \frac{(20)(10)\ (-53.13^0}{(10 + j15) + (6 - j8)}$

$= \frac{200\ (-53.13^0}{16 + j7} = \frac{200\ (-53.13^0}{17.46\ (23.62^0}$

$\therefore \quad I_1 = 11.45A$

Power consumed or dissipated in branch 1 $= I_1^2 R_1 = (11.45)^2\ (10)$

$= 1312.11W$

$I_2 = \frac{I \times Z_2}{Z_1 + Z_2} = \frac{(20)(10)\ (-53.13^0}{(10 + j15) + (6 - j8)}$

$= \frac{360.54\ (56.30^0}{17.46\ (23.62^0}$

$\therefore \quad I_2 = 20.64A$

Power dissipated in branch 2 $= I_1^2 R_2 = (20.64)^2\ (6)$

$= 2558.40W$

A circuit consists of a resistance of 10Ω, an inductance of 16 mH and a capacitance of 150 μF connected in series. A supply of 100 V at 50 Hz is given to the circuit. Find the current, power factor and power consumed by the circuit. Draw the vector diagram.

Sol.: R = 10Ω, L = 16 mH, C = 100 μF

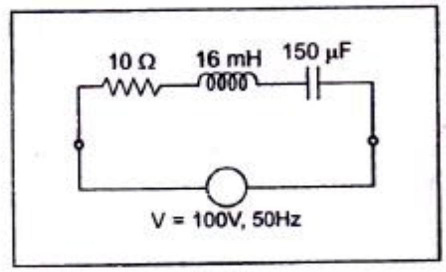


Fig. 3

$$X_L = 2\pi f L$$
$$= 2\pi \times 50 \times 16 \times 10^{-3}$$
$$= 5.02\Omega$$
$$X_C = \frac{1}{2\pi f C}$$
$$= \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}}$$
$$= 21.22\Omega$$

$$Z = R + j(X_L - X_C) = 10 + j(5.02 - 21.22)$$

$$Z = 10 - j\,16.2\ \Omega$$

$$\therefore \quad Z = 19.03 < -58.31^0\ \Omega$$

$$|Z| = \left|\frac{V}{I}\right|, \quad |I| = \left|\frac{V}{Z}\right| = \frac{100}{19.03} = 5.25 \text{ A}$$

Let current I be the reference phasor

$$I = 5.25 < 0^0 \text{ A}$$
$$V_R = I.R = (5.25)(10) = 52.5 < 0^0 \text{ volts}$$

$$V_L = I(jX_L)\ (5.25 < 0^0)(j\,5.02) = (5.25 < 0^0)(5.02 < 90^0)$$

$$= 26.35 < 90^0 \text{ volts}$$

$$V_C = (5.25 < 0^0)(-j\,21.22) = (5.25 < 0^0)(21.22 < -90^0)$$
$$= 111.40 < -90^0 \text{ volts}$$

The phasor diagram is shown in the Fig.4.

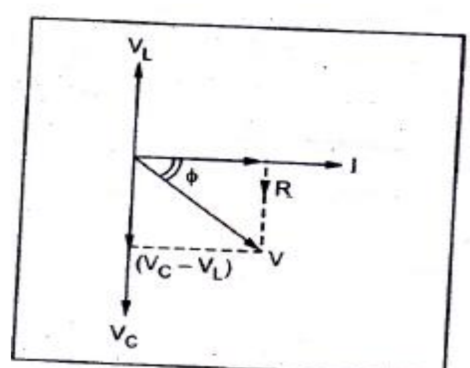


Fig. 4

$$\overline{V} = \overline{I}\,.\,\overline{Z}$$
$$= (5.25 < 0^0)(19.03 < -5831^0)$$
$$= 100 < -5831^0 \text{ volts}$$

As the circuit is capacitive current leads voltage by an angle of 58.31^0.

$\therefore$ p.f. = $\cos 58.31^0 = 0.525$ lead

Power = VI cos ϕ

= (100) (5.02) (0.525)
= 263.71 W

An inductive coil draws a current of 2A, when connected to a 230V, 50 Hz supply. The power taken by the coil is 100 watts. Calculate the resistance and inductance of the coil.

Sol. : Let r be resistance of coil while L Be inductance of coil.

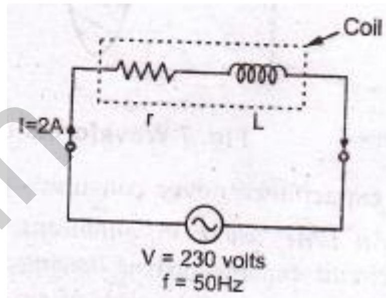


Fig. 9

$|Z| = V/I = 230/2 = 115($

Power taken by coil is consumed in resistive part of coil as inductance never consumes any part.

$P = I^2 r \quad \therefore r = P/I^{\uparrow}2 = 100/(2)^{\uparrow}2 = 25($

$\therefore r = 25 ($

Now $\quad Z = R + jX_L$

$|Z| = \sqrt{R^2 + X_L^2} \quad \therefore Z^2 = R^2 + X_L^2$

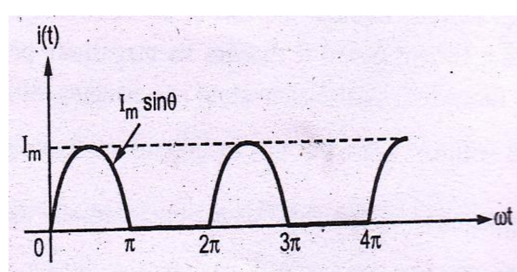
$X_L = \surd(Z^{\uparrow}2 - R^{\uparrow}2) = \surd((115)^{\uparrow}2 - (25)^{\uparrow}2) = 112.24($

But, $\quad X_L = 2\pi fL$

$\therefore \quad L = \dfrac{X_L}{2\pi f} = 112.\dfrac{24}{2\pi \times 50} = 0.3573H$

$\therefore \quad L = 0.3573$ H

Obtain the form factor of a half rectified sinewave.
Ans: The following waveform represented as,



I) R.M.S valve : Consider the first cycle,

$$I_{R.M.S} = \sqrt{\frac{\int_0^{2\pi} i^2(t)d\theta}{2\pi}} = \sqrt{\frac{\int_0^{\pi} i^2(t)d\theta + \int_{\pi}^{2\pi} i^2(t)d\theta}{2\pi}}$$

$$= \sqrt{\frac{\int_0^{\pi} I_m^2 \sin^2 \theta d\theta + 0}{2\pi}} \qquad = \frac{I_m}{\sqrt{2\pi}} \sqrt{\int_0^{\pi} \sin^2 \theta d\theta}$$

$$= \frac{I_m}{\sqrt{2\pi}} \times \sqrt{\frac{\pi}{2}} = \frac{I_m}{2} \qquad \ldots \int_0^{\pi} \sin^2 \theta d\theta = \frac{\pi}{2}$$

ii) Average value

$$I_{av} = \frac{\text{Area of 1 cycle}}{\text{base}} = \frac{\int_0^{2\pi} i(t)d\theta}{2\pi} = \frac{\int_0^{\pi} i(t)d\theta + \int_0^{2\pi} i(t)d\theta}{2\pi}$$

$$= \frac{\int_0^{\pi} I_m \sin \theta d\theta}{2\pi} \qquad = \frac{I_m}{2\pi} \int_0^{\pi} \sin \theta d\theta = \frac{I_m}{2\pi} [-\cos \theta]_0^{\pi}$$

$$= \frac{I_m}{2\pi} [-\cos\pi + \cos 0°] = \frac{2I_m}{2\pi} = \frac{I_m}{\pi}$$

iii) $K_f = \frac{R.M.S}{Average} = \frac{\frac{I_m}{2}}{\frac{I_m}{\pi}} = \frac{\pi}{2} = \mathbf{1.57}$

iv) $K_P = \frac{Maximum}{R.M.S} = \frac{I_m}{\frac{I_m}{2}} = 2$

b) A non inductive resistor of 10Ω is in series with a capacitor of 100 μF across a 250 volts, 50 Hz, A.C. supply. Determine the current taken by the capacitor and p.f. of the circuit. [5]

Ans. : The circuit is shown in the Fig. 4.

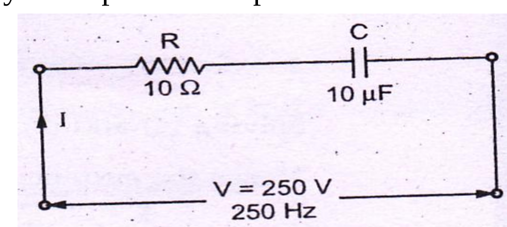


Fig.4

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$$

$= 31.8309\ \Omega$

∴ $Z = R - jX_C = 10 - j31.8309\ \Omega$

Let voltage applied be the reference .

∴ $I = \frac{V}{Z} = \frac{250 \angle 0°}{33.3647 \angle -72.559°} = 7.4929 \angle +72.559°$ A … Current

$\cos\phi = \cos(72.559°) = \mathbf{0.3\ leading}$ … Power factor

c) A current of average value 18.019 A is following in a circuit to which a voltage of peak value 141.42 V is applied. Determine i) Impedance in the polar form ii) Power Assume voltage lags current by 30° . **[5]**

Ans. : $I_{av} = 18.019$ A, $V_m = 141.42$ V , V lags I by 30°.

Assuming purely sinusoidal current and voltage waveforms,

$I = K_f \times I_{av} = 1.11 \times 18.019 = 20$ A … R.M.S. value

$V = \frac{V_m}{\sqrt{2}} = \frac{141.42}{\sqrt{2}} = 100$ V … R.M.S. value

Let voltage be reference as shown in the Fig. 5.

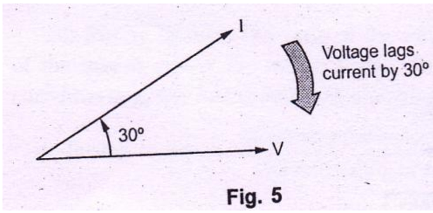


Fig. 5

$\therefore$ $V = 100 \angle 0°$ V

$I = 20 \angle +30°$ A

i) $Z = \frac{V}{I} = \frac{100 \angle 0°}{20 \angle 30°}$

$= 5 \angle -30°\ \Omega$ Polar form

ii) $P = VI \cos \emptyset = 100 \times 20 \times \cos(30)$

= 1732.0508 W

d) An a.c. current is given by I = 10 sin ωt + 3sin 3 ωt + 2 sin 5 ωt Find the r.m.s value of the current. [5]

Ans. : When a coil carries d.c. current and more than one alternating signals then the total heat produced is the sum of the heats produced by d.c. component and all the alternating components.

Let R = Resistance of wire

t = Time for which signals which signals are flowing

$\therefore$ $H_{total} = H_{dc} + H_1 + H_2 + \ldots$

$H_{total} = I_{rms}^2 \times R \times t$ where I_{rms} = Total r.m.s value

$H_{dc} = I_{dc}^2 \times R \times t$

$H_1 = I_{rms1}^2 \times R \times t$, $H_2 = I_{rms2}^2 \times R \times t$ …….

$\therefore$ $I_{rms}^2 \times R \times t = I_{dc}^2 Rt + I_{rms1}^2 Rt + I_{rms2}^2 Rt + \cdots$

$\therefore$ $I_{rms} = \sqrt{I_{dc}^2 + I_{rms1}^2 + I_{rms2}^2 + \cdots}$

For given example, $I_{dc} = 0$

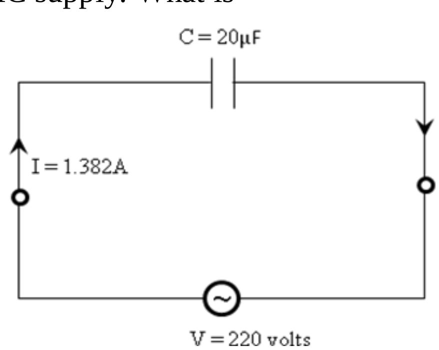
$I_{rms1} = \frac{I_{m1}}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07106$ A

$I_{rms2} = \frac{I_{m2}}{\sqrt{2}} = \frac{3}{\sqrt{2}} = 2.1213$ A

$I_{rms3} = \frac{I_{m3}}{\sqrt{2}} = \frac{2}{\sqrt{2}} = 1.4142$ A

$I_{rms} = \sqrt{0 + (7.07106)^2 + (2.1213)^2 + (1.4142)^2}$ = **7.5166 A**

current drawn by a pure capacitor of 20 μF is 1.382 A from 220V AC supply. What is the supply frequency?



sol. :

Given

C = 20 μF

V = 220volts

Now, $|Z| = \frac{V}{I} = \frac{220}{1.382} = 1.59.18\ \Omega$

Since it is a pure capacitor, its resistance is zero

$Z = -jX_c$ $\therefore |Z| = |X_c| = 159.18\ \Omega$

For capacitor, $X_c = \frac{1}{2\pi f C}$

$\therefore \quad f = \frac{1}{2\pi X_c C} = \frac{1}{2\pi \times 159.18 \times 20 \times 10^{-6}}$

$\therefore \quad f = \frac{1}{0.0200} = 50\text{Hz}$

$\therefore \quad f = 50\text{Hz}$

2) A parallel circuit comprises a resistor of 20 ohm in series with an inductive reactance of 15 ohm in one in one branch and a resistor of 30 ohm in series with a capacitive reactance of 20 ohm in the other branch. Determine the current and power dissipated in each branch if the total current drawn by the parallel circuit is 10 $\angle -30^0$ Amp**(8)**

Ans.: The arrangement is shown in the Fig.2. Ω= 25 ∠ 36.869^0 Ω
Z_2= 30 – j 20 Ω=36.055 ∠-33.69^0 Ω

By current division rule,

$I_1 = I_T \times \frac{Z_2}{Z_1 + Z_2} = \frac{10 \angle -30^0 \times 36.055 \angle 33.69^0}{20 + j15 + 30 - j20}$

$= \frac{360.055 \angle -63.90^0}{50.2493 \angle -5.71^0} = \mathbf{7.1752 \angle -57.98^0}$ **A**

And $I_2 = I_T \times \frac{Z_1}{Z_1 + Z_2} = \frac{10 \angle -30^0 \times 25 \angle 36.869^0}{50.2493 \angle -5.71^0}$ = **4.9752 + 12.579^0 A**

Only resistive part of each branch consumes the power given by $|I|^2$ R

$\therefore \quad P_1 = (I_1)^2 \times R_1 = (7.1752)^2 \times 20 =$ **1029.67 W.**

$\therefore \quad P_1 = (I_2)^2 \times R_2 = (4.9752)^2 \times 30 =$ **742.5784 W.**

EXPECTED QUESTIONS

1) ExplinGeneration of sinusoidal AC Voltage

2) Derive an Equation of Alternating E.M.F.

Define

a)Alternating quantity:.
b)Waveform:.
c)Instantaneous value:.
d) Alternation and cycle: cycle.
e) Periodic Time and Frequency:

3)DefineRoot-mean-square (R.M.S.) Value
4)DefineAverage Value
6) Show that power in a AC circuit containing pure ohmic resistance only VI
7) Show that power in a AC circuit containing containing pure inductance 0.
8) Show that power in a AC circuit containing containing pure capacitance 0.

9) Show that power **Series R-L circuit IS VIcosΦ**

10) Define Real power, Reactive Power, Apparent power and power Factor

11) Show that power **Series R – C circuit is VIcosΦ**

12) Show that power **Series R-L –C circuit IS VIcosΦ**

SOLUTION TO QUESTION BANK

3) Derive an expression for the instantaneous power in a pure conductor energised by sinusoidal voltage. Draw the waveshapes of voltage, current and power signals involved.

Sol. : A.C. through pure capacitors:

Consider a simple circuit consisting of a pure capacitor of c-farads, connected across a voltage given by the equation, $v = V_m \sin\omega t$. The circuit is shown in Fig. 4.

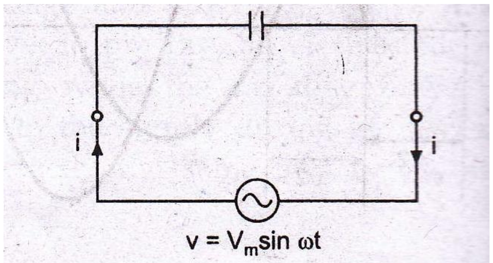


Fig. 4 Purely capacitive circuit

The current i charges the capacitor c. The instantaneous charge 'q' on the plates of the Capacitor is given by

$$q = Cv$$

$$q = CV_m \sin(\omega t)$$

Now, current is rate of flow of charge,

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin\omega t)$$

$$i = CV_m \frac{d}{di}(\sin\omega t) = CV_m \omega\cos\omega t$$

$$i = \frac{V_m}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

Where $I_m = \frac{V_m}{X_c}$

Where

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}\,\Omega$$

The above equation clearly shows that the current is purely sinusoidal and having . phase angle $+\frac{\pi}{2}$ radians i.e. $+ 90^0$.

This means current leads voltage applied by 90^0. The positive sign indicates leading nature of the current. If current is assumed reference, we can say that voltage across capacitor passing through the capacitor by 90^0.

Fig. 5 shows waveforms of voltage applied by 90^0. The current and the corresponding phasor diagram. The current waveform starts earlier by 90^0 in comparison with voltage waveform. When voltage zero, the current has positive maximum value.

In purely capacitive circuit, current leads voltage by 90^0.

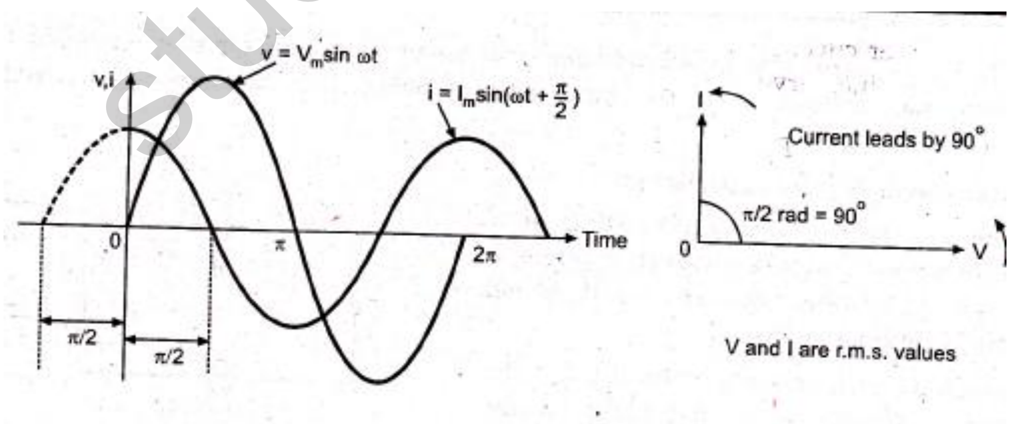


Fig. 5

4) A parallel circuit comprises a resistor of 20 ohm in series with an inductive reactance of 15 ohm in one in one branch and a resistor of 30 ohm in series with a capacitive reactance of 20 ohm in the other branch. Determine the current and powerdissipated in each branch if the total current drawn by the parallel circuit is $10\angle -30^0$ Amp **(8)**

Ans.: The arrangement is shown in the Fig.2.

$Z_1 = 20 + j\,15\ \Omega = 25 \angle 36.869^0\ \Omega$

$Z_2 = 30 - j\,20\ \Omega = 36.055 \angle -33.69^0$

Ω

By current division rule,

$I_1 = I_T \times \frac{Z_2}{Z_1 + Z_2} =$

$$\frac{10 \angle -30^0 \times 36.055 \angle 33.69^0}{20 + j15 + 30 - j20}$$

$$= \frac{360.055 \angle -63.90^0}{50.2493 \angle -5.71^0} = \mathbf{7.1752 \angle -57.98^0\ A}$$

And $I_2 = I_T \times \frac{Z_1}{Z_1 + Z_2} = \frac{10 \angle -30^0 \times 25 \angle 36.869^0}{50.2493 \angle -5.71^0} = \mathbf{4.9752 + 12.579^0\ A}$

Only resistive part of each branch consumes the power given by $(I)^2\ R$

$\therefore$ $P_1 = (I_1)^2 \times R_1 = (7.1752)^2 \times 20 = \mathbf{1029.67\ W.}$

$\therefore$ $P_1 = (I_2)^2 \times R_2 = (4.9752)^2 \times 30 = \mathbf{742.5784\ W.}$

5) With a neat sketch briefly explain how an alternating voltage is produced when a coil is rotated in a magnetic field.

Sol: Generation of A.C. voltage :

The machines which are used to generate electrical voltages are called generators. The generators which generate purely sinusoidal a.c. voltages are called alternators.

The basic principle of an alternator the principle of electromagnetic induction. We have discussed earlier the effect of electromagnetic induction and Faraday's Law of governing this phenomenon. So, sine wave is generated according to Faraday's Law of electromagnetic induction. It says that, whenever there is a relative motion between the conductor and the magnetic field in which it is kept, an e.m.f. gets induced in the conductor. The relative motion may exist because of movement of conductors with respect to magnetic field or movement of magnetic field with respect to conductor. Such an induced e.m.f. then can be used to supply the electrical load.

Let us see how an alternator produces a sine wave, with the help of simplest form of an alternator called single turn or single loop alternator.

Single turn generator:

Construction: It consists of a permanent magnet of two poles. A single turn rectangular coil is kept in the vicinity of the permanent magnet. The coil made up of same conducting material like copper or aluminium. The coil is made up of two conductors namely a-b and c-d. such two conductors are connected at one end to form a coil. This is shown in fig.2.

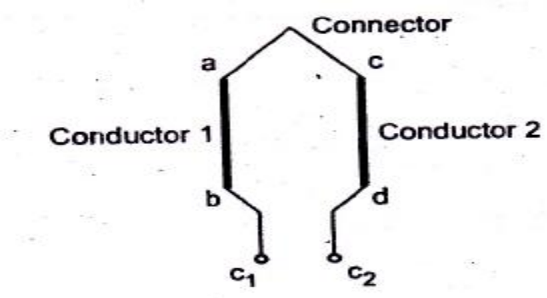


Fig. 2 Single turn coil

The coil is so placed that it can be rotated about its own axis in clockwise or anticlockwise direction. The remaining two ends C1 and C2 of the coil are connected to the rings mounted on the shaft called slip rings are also rotating members of the alternator. The two brushes P and Q are resting on the slip rings. The brushes are stationary and making contact with the slip rings. The slip rings and brush assembly is necessary to collect the current induced in the rotating coil and make it available to the stationary external resistance. The overall construction is shown in Fig.3.

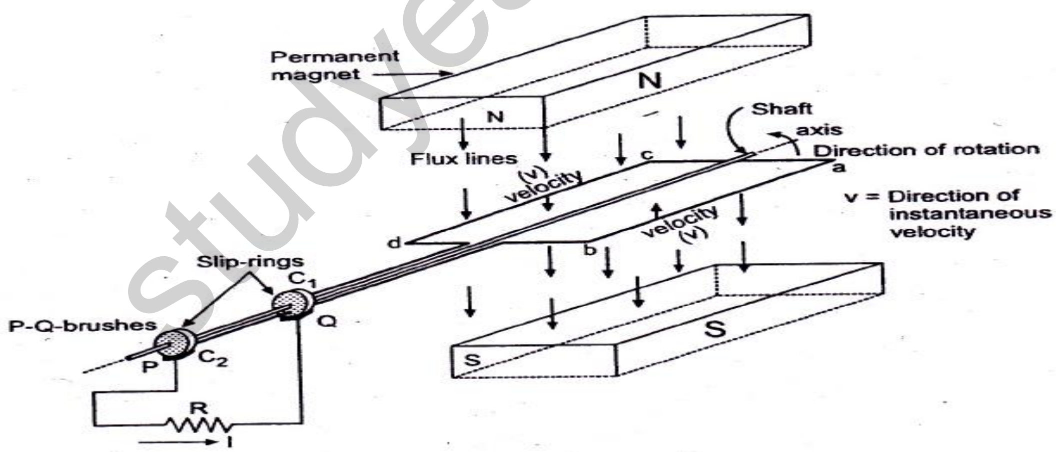


Fig. 3 Single turn generator

Working:

The coil is rotated in anti clockwise direction. While rotating, the conductors ab and cd cut the lines of flux of the permanent magnet. Due to Faraday's law of electromagnetic induction, an e.m.f. gets induced in the conductors. This e.m.f. drives a current through resistance R connected across the brushes P and Q. The magnitude of the induced e.m.f. depends on the position of the coil in the magnetic field. Let us see the relation between magnitude of the induced e.m.f. and the positions of the coil. Consider different instants and the different positions of the coil.

Instant 1 : Let the initial position of the coil be as shown in Fig. 4. The plane of the coil is perpendicular to the direction of the magnetic field. The instantaneous component of velocity of conductors ab and cd, is parallel to the magnetic field as shown and there cannot be the cutting of the flux lines by the conductors. Hence, no e.m.f. will be generated in the conductors ab and cd and not current will flow through the external resistance R. this position can be represented by considering the front view of the Fig. 4 as shown in Fig. 4(a).

The angle θ is measured from plane of the magnetic flux.

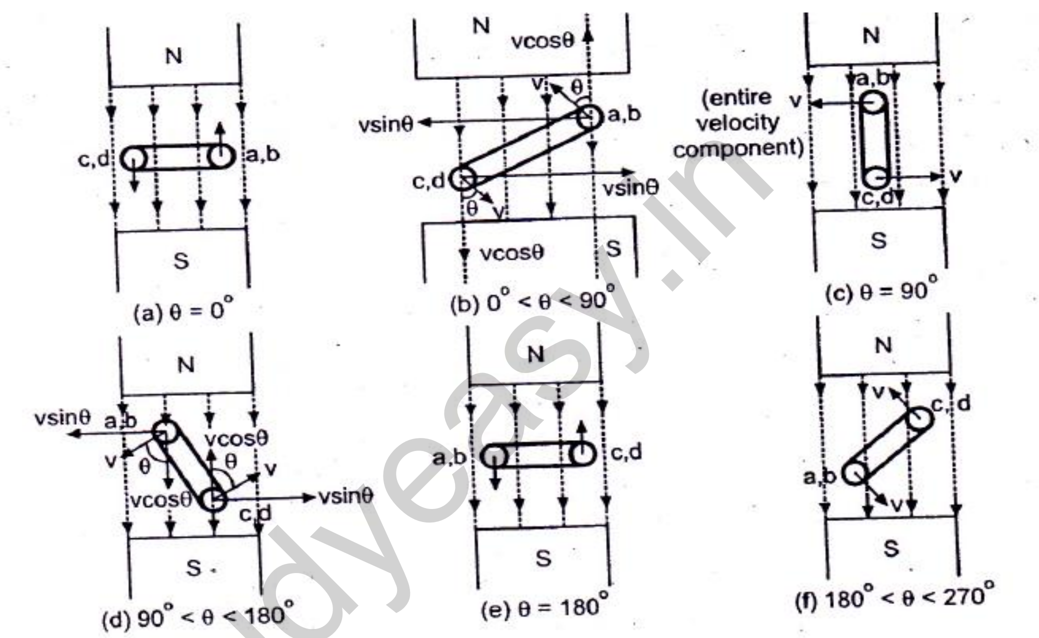


Fig. 4 The different instants of induced e.m.f. (Front view)

Instant 2 : When the coil is rotated in anticlockwise direction through some angle θ, then the velocity will have two components v sin θ perpendicular to flux lines and v cos θ parallel to the flux lines. Due to v sin θ component, there will be cutting of the flux and proportionally, there will be induced e.m.f. in the conductors ab and cd. This e.m.f. will drive a current through the external resistance R. This is shown in Fig.4(b).

6) Derive an expression for the instantaneous power in a pure conductor energised by sinusoidal voltage. Draw the waveshapes of voltage, current and power signals involved.

Sol. : A.C. through pure capacitors:

Consider a simple circuit consisting of a pure capacitor of c-farads, connected across a voltage given by the equation, $v = V_m \sin\omega t$. The circuit is shown in Fig. 4.

The current i charges the capacitor c.

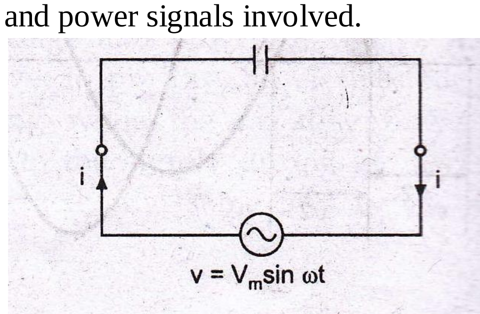


Fig. 4 Purely capacitive circuit

The instantaneous charge ‘q’ on the plates of the Capacitor is given by

$$q = Cv$$

$$q = CV_m \sin(\omega t)$$

Now, current is rate of flow of charge,

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin\omega t)$$

$$i = CV_m \frac{d}{di}(\sin\omega t) = CV_m \omega\cos\omega t$$

$$i = \frac{V_m}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

Where $I_m = \frac{V_m}{X_c}$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}\,\Omega$$

The above equation clearly shows that the current is purely sinusoidal and having phase angle $+\frac{\pi}{2}$ radians i.e. $+90^0$.

This means current leads voltage applied by 90^0. The positive sign indicates leading nature of the current. If current is assumed reference, we can say that voltage across capacitor passing through the capacitor by 90^0.

Fig. 5 shows waveforms of voltage applied by 90^0. The current and the corresponding phasor diagram. The current waveform starts earlier by 90^0 in comparison with voltage waveform. When voltage zero, the current has positive maximum value.

In purely capacitive circuit, current leads voltage by 90^0.

Please refer fig. 5 on next page.

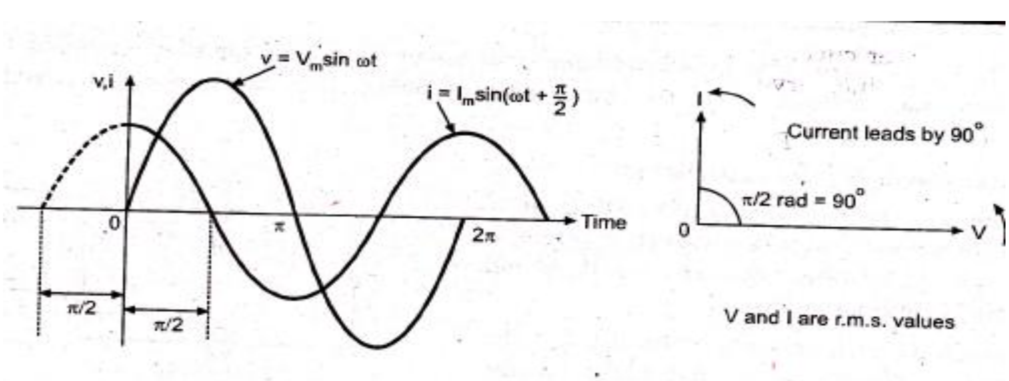


Fig. 5

Concept of capacitive reactance :

We have seen while expressing current equation in the standard form that

The term is called capacitive reactance and is measured in ohms.

So capacitive reactance is defined as the opposition offered by the capacitance of a circuit to the flow of an alternating sinusoidal current.

X_c is measured in ohms and it depends on the frequency of the applied voltage.

The capacitive reactance is inversely proportional to the Frequency for constant C.

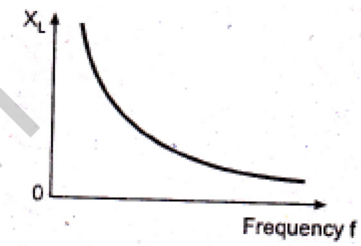


Fig. 6

$$X_c \propto \frac{1}{f} \quad \text{for constant C}$$

The graph of X_c Vs F is a rectangular hyperbola as shown in fig. 6.

If the frequency is zero, which is so for d.c. voltage, the capacitive reactance is Infinity. Therefore , it is said that the capacitance offers open circuit to the d.c. or it block d.c.

Power: The expression for the instantaneous power can be obtained by talking the product of instantaneous voltage and current.

$$P = v \times i = V_m \sin(\omega t) \times I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin(\omega t)\cos(\omega t) \quad \text{as } \sin\left(\omega t + \frac{\pi}{2}\right) = \cos\omega t$$

$$P = \frac{V_m I_m}{2}\sin(2\omega t) \quad \text{as } 2\sin\omega t\cos\omega t = \sin 2\omega t$$

Thus, power curve is a shine wave of frequency double that of applied voltage. The average value sine curve over a complete cycle is always zero.

$$P_{av} = \int_0^{2\pi} \frac{V_m I_m}{2} \sin(2\omega t)\, d(\omega t) = 0$$

The fig. 7 shows waveforms of current, voltage and power. It can be observed from the figure that when power curve is positive, in practice, an electrostatic energy gets stored in the capacitor during its changing while the negative power curve represents that the energy stored is turned back to the supply during its discharging. The areas of positive and negative loops are exactly the same and hence, average power consumption is zero.

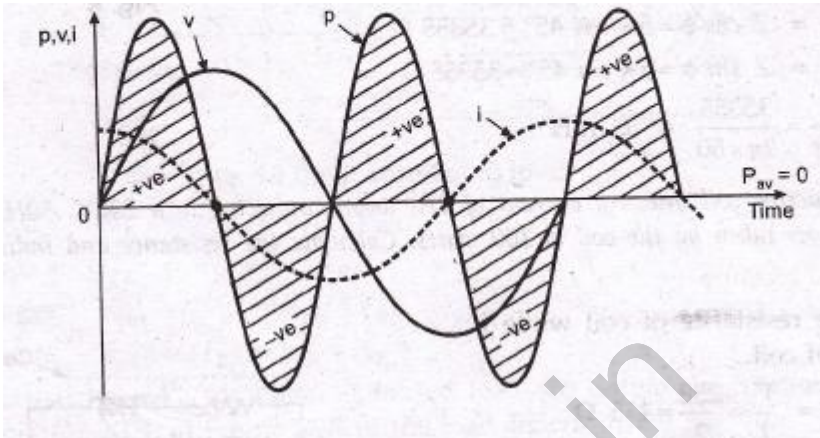


Fig. 7 Waveforms of voltage, current and power

Pure capacitance never consumes power.

7) An EMF whose instantaneous value is 100 sin (314t – π/4) volts is applied to a circuit and the current flowing through it is 20sin(314t-15708) amperes. Find the frequency and the values of circuit elements, assuming a series combination of circuit elements.

Sol. : $V = 100 \sin(314t - \pi/4)$ volts

$I = 20\sin(314t - 15708)$ A

$V_m = 100$ volts, $\quad V_{rms} = \frac{I_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71\ volts$

$I_m = 20A, \quad I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 14.1421$ A

Comparing $V=100 \sin(314t - \pi/4)$ with $V = V_m \sin(\omega t + \emptyset)$

We have

$\omega = 314 \qquad \therefore \quad 2\pi f = 314$

$$f = \frac{314}{2\pi} \approx 50\text{Hz}$$

$$V = 70.71\angle -45^0 \text{ volts}$$

$I = 14.1421 \angle -90^0$ A $\qquad$ as 1.5708 rad≡ 90^0

Angle between V and I = $90^0 - 45^0$

$\therefore \quad \emptyset = 45^0, \qquad \cos \emptyset = 0.7071$

Now, $Z = \frac{V}{I} = \frac{70.71}{14.1421} = 5$

As the current is lagging behind the voltage

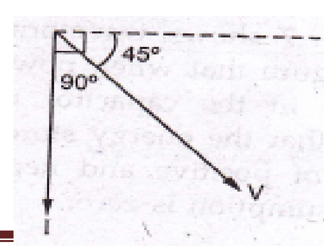


Fig. 8

the circuit is inductive consisting of resistance and inductance reactance.

$$R = Z \cos \phi = 5 \times \cos 45^0 = 3.5355\Omega$$

$$X_L = Z \sin \phi = 5 \times \sin 45^0 = 3.5355\Omega$$

$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{3.5355}{2\pi \times 50} = 11\text{mH}$$

8) An inductive coil draws a current of 2A, when connected to a 230V, 50 Hz supply. The power taken by the coil is 100 watts. Calculate the resistance and inductance of the coil.

Sol. : Let r be resistance of coil while L Be inductance of coil.

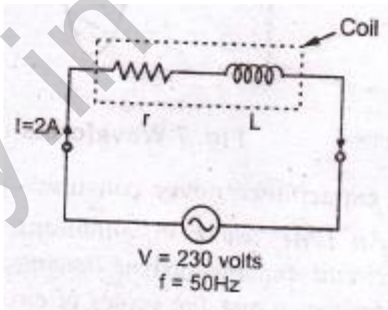


Fig. 9

$$|Z| = \frac{V}{I} = \frac{230}{2} = 115\Omega$$

Power taken by coil is consumed in resistive part of coil as inductance never consumes any part.

$P = I^2 r \qquad \therefore r = \frac{P}{I^2} = \frac{100}{(2)^2} = 25\Omega$

$\therefore$ r = 25 Ω

Now $\qquad Z = R + jX_L$

$$|Z| = \sqrt{R^2 + X_L^2} \qquad \therefore Z^2 = R^2 + X_L^2$$

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(115)^2 - (25)^2} = 112.24\Omega$$

But, $\qquad X_L = 2\pi f L$

$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{112.24}{2\pi \times 50} = 0.3573\text{H}$$

$\therefore \qquad L = 0.3573$ H

9) Derive expressions for average value and RMS value of a sinusoidally varying AC voltage.

Sol : Analytical method to find rms value of alternating quantity.

Consider sinusoidally varying alternating current and square of this current as shown in

Fig. 5.

Please refer Fig.5 on next page.

The current $\qquad I = I_m \sin \theta$

while square of current

$$i_2 = I_m^2 \ sin^2\theta$$

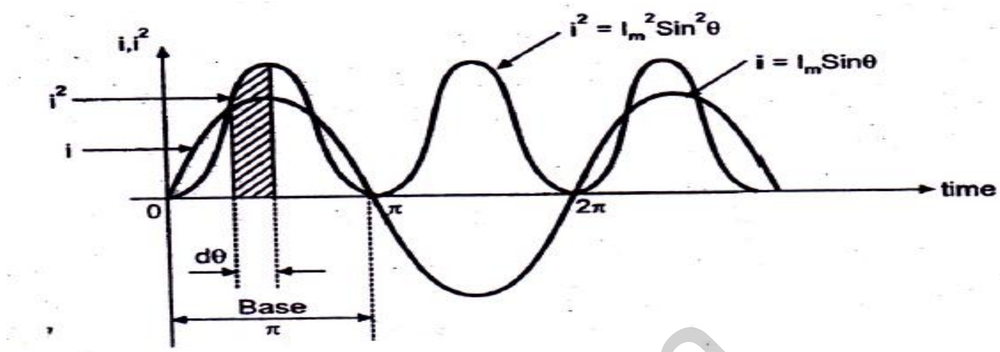


Fig. 5 Waveform of current and square of the current

Area of curve over half a cycle be calculated by considering an interval dθ as shown.
Area of square curve over half cycle = $\int_0^\pi i^2\, d\theta$ and length of the base is π

∴ Average value of square of the current over half cycle

$$= \frac{\text{Area of curve over half cycle}}{\text{Length of base over half cycle}} = \frac{\int_0^\pi i^2\, d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^\pi i^2\, d\theta = \frac{1}{\pi}\int_0^\pi I_m^2 \ \sin^2\ d\theta$$

$$= \frac{I_m^2}{\pi}\left[\frac{1-cos2\theta}{2}\right] d\theta$$

$$= \frac{I_m^2}{\pi}\left[\theta - \frac{sin2\theta}{2}\right]_2^\pi = \frac{I_m^2}{2\pi}\ [\pi]$$

$$= \frac{I_m^2}{2}$$

Hence, root mean square value i.e. r.m.f. value can be calculated as,

$$I_{r.m.f.} = \sqrt{\text{mean or average of square of current}} = \sqrt{\frac{I_m^2}{2}}$$

$$\frac{I_m}{\sqrt{2}}$$

$$I_{r.m.f.} = 0.707\ I_m$$

The r.m.s. value of the sinusoidal alternating current is 0.707 times the maximum or peak value or amplitude of that alternating current.

Importance of R.M.S value :

In case of alternating quantities, the values are used for specifying magnitudes of alternating quantities.

In practice, everywhere, r.m.f. values are used to analyze alternating quantities. The ammeters and voltmeters record the r.m.f. value of current and voltage respectively.

9)Average value :

The average value of an alternating quantity is defined as that value which is obtained by averaging all the instantaneous values over a period of half cycle.

For a symmetrical a.c, the average value over a complete cycle is zero as both positive and negative half cycle are exactly identical. Hence, the average value is defined for half cycle only.

Average value can also be expressed by that steady current which transfers across any circuit, the same amount of charge as is transferred by that alternating current during the same time. The average value for sinusoidally varying alternating current can be obtained by,

1) Graphical Method and 2) Analytical method

Analytical method:

For an unsymmetrical a.c., the average value must be obtained for one complete cycle but symmetrical a.c. like sinusoidal, it is to be obtained for half cycle.

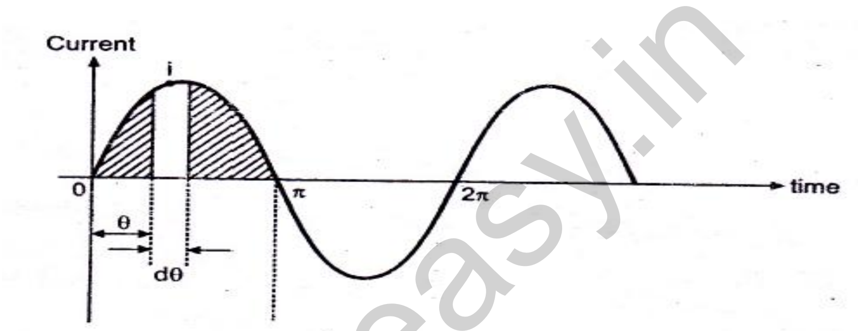


Fig. 6 Average value of an alternating current

Consider sinusoidal varying current, $I = I_m \sin\theta$.

Consider the elementary interval of instant 'dθ' as shown in Fig.6. The average instantaneous value of current in this interval is say, 'i' as shown.

The average value can obtained by taking ratio of area under cure over half cycle to length of the base for half cycle.

$$\therefore \quad I_{av} = \frac{\text{Area of curve over half cycle}}{\text{Length of base over half cycle}} = \frac{\int_0^{\pi} i\, d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} i^2\, d\theta = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin\theta d\theta = \frac{I_m}{\pi}\int_0^{\pi} \sin\theta$$

$$= \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_m}{\pi}\left[-\cos\pi + \cos 0\right]$$

$$= \frac{I_m}{\pi}[2] = \frac{2I_m}{\pi}$$

$$\therefore \quad I_{av} = \frac{2}{\pi} I_m$$

i.e. $\quad I_{av} = 0.637 I_m$

Similarly, $\quad V_{av} = 0.637 V_m$

The average value is used for application like battery charging, etc. It is rarely used in practice.

10) A circuit having a resistance of 12Ω, an inductance of 0.15 H and a capacitance of

100μF in series is connected across a 100V, 50 Hz supply. Calculate the impedance, current, the phase difference between the current and supply voltage.

Sol:

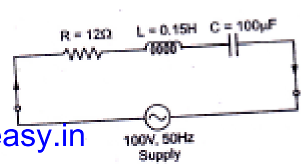


Fig. 7

$X_L = 2\pi FL = 2\pi \times 50 \times 0.15$
$= 100 \times \pi \times 0.15$
$\therefore X_L = 47.12\Omega$
$X_C = \frac{1}{2\pi fL} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}$
$\therefore X_C = 31.83\Omega$

$Z = R + jX_L - jX_C = 12 + j47.12 - j31.83$
$= 12 + j15.29\Omega$
$V = 100\text{volts} = 100\angle 0^0$
$Z = 12 + j15.29 = 19.43 \angle 51.87^0\Omega$
$I = \frac{V}{Z} = \frac{100\angle 0^0}{19.43\angle 51.87^0} = 5.146\angle 51.87^0\Omega A$
$\therefore \quad I = 5.146A$
p.f. = $\cos\phi = \cos 51.87^0 = 0.6174$ lag, $\quad \phi = 51.87^0$

11) Two circuits with impedances of Z1 = 10 + j15Ω and Z2 = 6-j8Ω are connected in parallel. If the supply current is 20A, what is the power dissipated in each branch?

Sol :

$Z_1 = 10 + j15\Omega = 18.027 \angle 56.30^0\Omega$
$Z_2 = 6\text{-}j8\Omega = 10 \angle 53.13^0\Omega$
$I = 20A$
$I_1 = \frac{I \times Z_2}{Z_1 + Z_2} =$
$= \frac{200\angle -53.13^0}{16 + j7} = \frac{200\angle -53.13^0}{17.46\angle 23.62^0}$
$\therefore \quad I_1 = 11.45A$

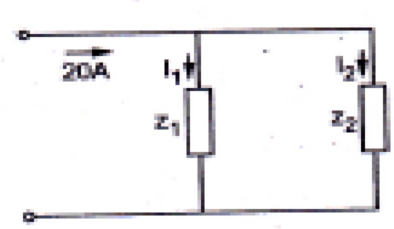


Fig. 8

Power consumed or dissipated in branch 1 = $I_1^2 R_1 = (11.45)^2$ (10
= 1312.11

$I_2 = \frac{I \times Z_1}{Z_1 + Z_2} =$
$= \frac{360.54\angle 56.30^0}{17.46\angle 23.62^0}$
$\therefore \quad I_2 = 20.64A$

Power dissipated in branch 2 = $I_1^2 R_2 = (20.64)^2$ (6)
= 2558.40W

Unit-3: Three Phase Circuits

Necessity and advantages of three phase systems, generation of three phase power, definition of Phase sequence, balanced supply and balanced load. Relationship between line and phase values of balanced star and delta connections. Power in balanced three-phase circuits, measurement of power by two-wattmeter method. Illustrative examples

3.23 Necessity and Advantages of three-phase systems.

In a single phase a.c. generation, a number of armature coils are connected to from one winding, which when rotated in a magnetic field generates a voltage called single-phase voltage. But it is found that in many applications, the single-phase system is not very satisfactory. For instance, a single-phase induction motor is not self-starting unless it is fitted with an anxiliary winding. Hence it is found that a three-phase induction motor is more suitable as it is self-starting and has better efficiency and power factor than its single-phase counterpart.

The advantages of three phase systems are as follows:

1. The output of a three-phase machine is always greater than that of a single-phase machine of the same size of frame. So, for a given size and voltage, a three-phase machine (e.g. a 3-phase alternator) occupies less space and costs less than the single-phase machine having the same rating.
2. To transmit and distribute a given amount of power over a given distance, a three- phase transmission line requires less copper than a single-phase line.
3. Three-phase motors have an absolutely uniform torque, whereas single-phase motors (except commutator motors) have a pulsating torque.
4. Single-phase motors (except commutator motors) are not self-starting. Three-phase motors are self-starting.
5. The pulsating nature of the armature reaction in single-phase alternators causes difficulty with parallel running unless the poles are fitted with exceedingly heavy dampers. Three-phase generators work in parallel without difficulty.
6. The connection of single-phase generators in parallel gives rise to harmonics, whereas three-phase generators can be conveniently connected with causing generation of harmonics.
7. In the case of a three-phase star system, two different voltages can be obtained, one between lines and the other between line and phase, whereas in the case of a single phase system only one voltage can be obtained.
8. In a single-phase system, the instantaneous power is a function of time and hence fluctuates with respect to time. This fluctuating power causes considerable vibrations in single-phase motors. Hence performance of single-phase motors in poor; Whereas instantaneous power in a symmetrical three-phase system is constant.

3.24 Generation of 3-phase E.M.F.

In the 3-phase system, there are three equal voltages of the same frequency but displaced from one another by 120^0 electrical. These voltages are produced by a three-phase generator which has three identical windings or phases displaced 120^0 electrical apart. When these windings are rotated in a magnetic field, e.m.f. is induced in each winding or phase. These e.m.f. s are of the same magnitude and frequency but are displaced from one another by 120^0 electrical.

Consider three electrical coils a_1a_2, b_1b_2 and c_1c_2 mounted on the same axis but displaced from each other by 120^0 electrical. Let the three coils be rotated in an anticlockwise direction in a bipolar magnetic field with an angular velocity of ω radians/sec, as shown in Fig. 3.80. Here, a_1, b_1 and c_1 are the start terminals and a_2, b_2 and c_2 are the end terminals of the coils.

When the coil a_1a_2 is in the position AB shown in Fig. 3.80, the magnitude and direction of the e.m.f. s induced in the various coils is as under:

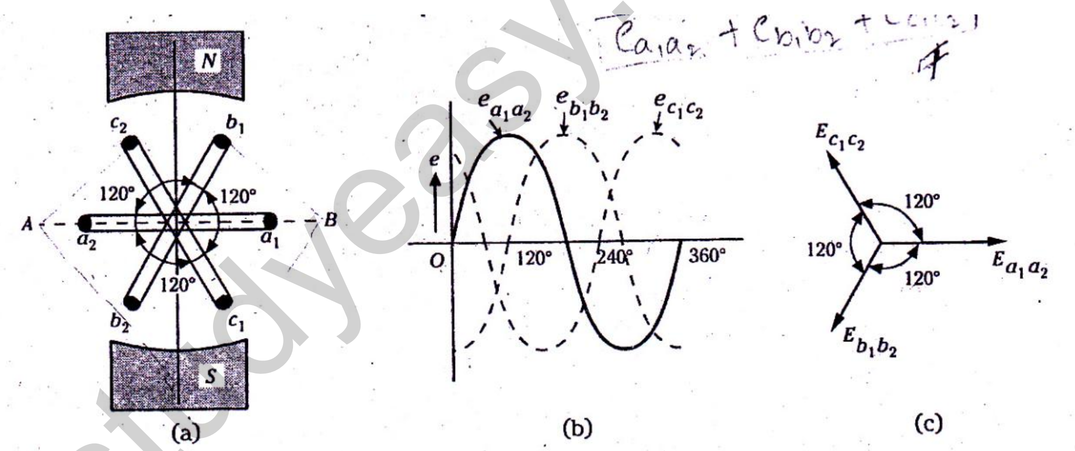


Fig. 3.80

a) E.m.f. induced in coil a_1a_2 is zero and is increasing in the positive direction. This is indicated by $e_{a1\ a2}$ wave in Fig. 3.80 (b).

b) The coil b_1b_2 is 120^0 electrically behind coil a_1a_2. The e.m.f. induced in this coil is negative and is approaching maximum negative value. This is shown by the e_{b1b2} wave.

c) The coil c_1c_2 is 240^0 electrically behind a_1a_2 or 120^0 electrically behind coil b_1b_2. The e.m.f. induced in this coil is positive and is decreasing. This is indicated by wave $e_{c1\ c2}$.

Thus, it is apparent that the e.m.f. 's induced in the three coils are of the same magnitude and frequency but displaced 120^0 electrical from each other.

Vector Diagram: The r.m.s. values of the three phase voltage are shown vectorially in Fig. 3.80(c).

Equations: The equations for the three voltages are:

$$e_{a1\,a2} = E_m \sin \omega t$$

$$e_{b1\,b2} = E_m \sin \omega t \quad ; \quad e_{c1\,c2} = E_m \sin \omega t$$

3.25 Meaning of phase sequence

The order in which the voltages in the voltages in the phases reach their maximum positive values is called the phase sequence. For example, in Fig. 3.80(a), the three coils a_1a_2, b_1b_2 and c_1c_2 are rotating in anticlockwise direction in the magnetic field. The coil a_1a_2 is 120^0 electrical ahead of coil b_1b_2 and 240^0 electrical ahead of coil c_1c_2. Therefore, e.m.f. in coil a_1a_2 leads the e.m.f. in coil b_2b_2 by 120^0 and that in coil c_1c_2 by 240^0. It is evident from Fig. 3.80(b) that $e_{a1\,a2}$ attains maximum positive first, then $e_{b1\,b2}$ and $e_{c1\,c2}$. In other words, the order in which the e.m.f. s in the three phases a_1a_2 , b_1b_2 and c_1c_2 attain their maximum positive values is a,b,c. Hence, the phase sequence is a,b,c.

3.25.1 Naming the phases

The 3 phases may be numbered (1,2,3) or lettered (a,b,c) or specified colours (R Y B). By normal convention, sequence RYB is considered positive and R B Y negative.

3.26 25 Meaning of phase sequence

It is necessary to employ some systematic notation for the solution of a.c. circuits and systems containing a number of e.m.f. s. acting and currents flowing so that the process of solution is simplified and less prone to errors.

It is normally preferred to employ double-subscript notation while dealing with a.c. electrical circuits. In this system, the order in which the subscripts are written indicates the direction in which e.m.f. acts or current flows.

For example, if e.m.f. is expressed as E_{ab}, it indicates that e.m.f. acts from a to b; if it is expressed as E_{ba}, then the e.m.f. acts in a direction opposite to that in which E_{ab} acts. (Fig. 3.81) i.e., $E_{ba} = - E_{ab}$.

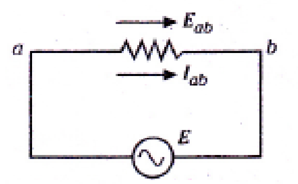


Fig. 3.81

Similarly, I_{ab} indicates that current flows in the direction from a to b but I_{ba} indicates that current flows in the direction from b to a; i.e., $I_{ba} = -I_{ab}$.

3.27 Balanced Supply and Load

When a balanced generating supply, where the three phase voltages are equal, and the phase difference is 120^0 between one another, supplies balanced equipment load, where the impedance of the three phases or three circuit loads are equal, then the current flowing through these three phases will also be equal in magnitude, and will also have a phase difference of 120^0 with one another. Such an arrangement is called a balanced load.

3.28 Obtaining Relationship between Line & Phase Values & Expression for power for Balanced Star Connection

This system is obtained by joining together similar ends, either the start or the finish; the other ends are joined to the line wires, as shown in Fig.3.82(a). the common point N at which similar (start or finish) ends are connected is called the neutral or star point. Normally, only three wires are carried to the external circuit, giving a 3-phase, 3-wire, star-connected system; however, sometimes a fourth wire known as neutral wire, is carried to the neutral point of the external load circuit, giving a 3-phase, 4-wire connected system.

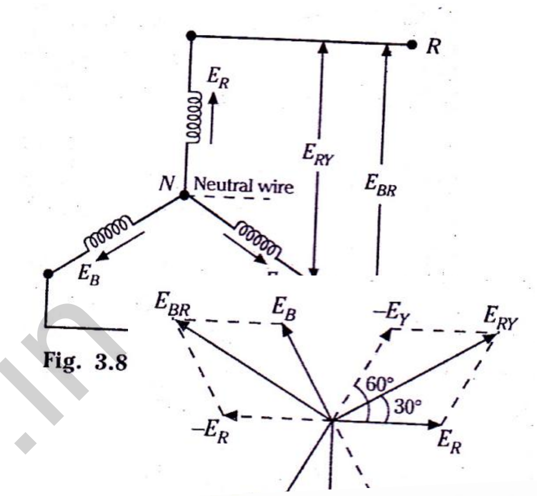


Fig. 3.82 (b) Vector Diagram of Line and Phase voltages. 3-Phase Star-Connected System

E_{YB}

The voltage between any line and the neutral point, i.e., voltage across the phase winding, is called the phase voltage; while the voltage between any two outers is called line voltage. Usually, the neutral point is connected to earth.

In Fig.3.82(a), positive directions of e.m.f.s. are taken star point outwards. The arrow heads on e.m.f.s. and currents indicate the positive direction. Here, the 3-phases are numbered as usual: R,Y and B indicate the three natural colours red, yellow and blue respectively.

By convention, sequence RYB is taken as positive and RYB as negative.

In Fig.3.82(b), the e.m.f.s induced in the three phases, are shown vectorially. In a star-connection there are two windings between each pair of outers and due to joining of similar ends together, the e.m.f.s induced in them are in opposition.

Hence the potential difference between the two outers, know as line voltage, is the vector difference of phase e.m.f.s of the two phases concerned.

For example, the potential difference between outers R and Y or Line voltage E_{RY}, is the vector difference of phase e.m.f.s E_R and E_Y or vector sum of phase e.m.f.s E_R and ($-E_Y$).

i.e. $E_{RY} = E_R - E_Y$ (vector difference)

or $E_{RY} = E_R + (-E_Y)$ (vector sum)

as phase angle between vectors E_R and ($-E_Y$) is 600,

∴ from vector diagram shown in Fig.3.82(b),

$$E_{RY} = \sqrt{E_R^2 + E_Y^2 + 2E_R E_Y \cos 60^0}$$

Let $E_R = E_Y = E_B = E_P$ (phase voltage)

Then line voltage $E_{Ry} = \sqrt{E_P^2 + E_P^2 + (2E_P E_P \times 0.5)} = \sqrt{3}\, E_P$

Similarly, potential difference between outers Y and B or line. Voltage $E_{YB} = E_Y - E_B = \sqrt{3}\, E_P$ and potential difference between outers B and R, or line voltage $E_{BR} = E_B - E_R = \sqrt{3}\, E_P$.

In a balanced star system, E_{RY}, E_{YB} and E_{BR} are equal in magnitude and are called line voltages.

∴ $E_L = \sqrt{3}\, E_P$

Since, in a star-connected system, each line conductor is connected to a separate phase, so the current flowing through the lines and phases are the same.

i.e. Line current I_L = phase current I_P

if the phase current has a phase difference of ϕ with the voltage,

power output per phase = $E_P I_P \cos\phi$

total power output, $P = 3E_P I_P \cos\phi$

$= 3\frac{E_L}{\sqrt{3}} I_P \cos\phi$

$= \sqrt{3}\, E_L\, I_L \cos\phi$

i.e. power = $\sqrt{3}$ x line voltage x line current x power factor

apparent power of 3-phase star-connected system

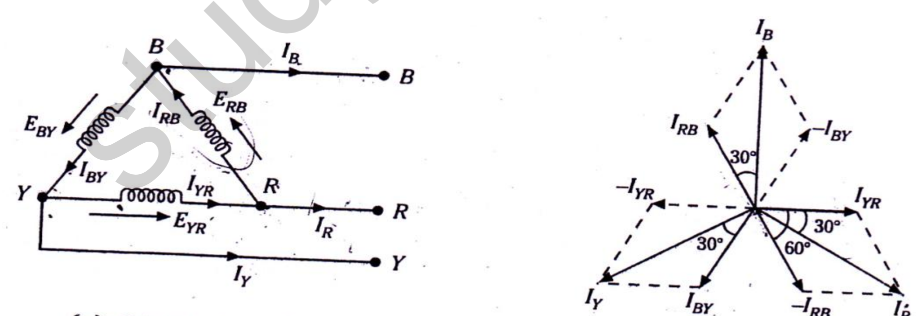
= 3 x apparent power per phase

$= 3E_P I_P = 3\text{x} \frac{E_L}{\sqrt{3}} \text{ x } I_L = \sqrt{3}\, E_L\, I_L$

3.29 Obtaining Relationship between Line and Phase Values and Expression for Power for Balanced Delta Connection

When the starting end of one coil is connection to the finishing end of another coil, as shown in Fig.3.83(a), delta or mesh connection is obtained. The direction of the e.m.f.s is as shown in the diagram.

From Fig.3.83 it is clear that line current is the vector difference of phase currents of the two phases concerned. For example, the line current in red outer I_R will be equal to the vector difference of phase currents I_{YR} and IRB. The current vectors are shown in Fig.3.83(b).



(a) Connection Diagram (b) Vector Diagram

Fig. 3.83

Referring to Fig.3.83(a) and (b),

Line current, $I_R = I_{YR} - I_{RB}$ (vector difference)

$= I_{YR} + (-I_{RB})$ (vector sum)

As the phase angle between currents I_{YR} and $-I_{RB}$ is 600

∴ $I_R = \sqrt{I_{YR}^2 + I_{RB}^2 + 2I_{YR} I_{RB} \cos 60^0}$

For a balanced load, the phase current in each winding is equal and let it be = I_P.

∴ Line current, $I_R = \sqrt{I_{YR}^2 + I_{RB}^2 + 2I_P I_P \text{x} 0.5} = \sqrt{3}\, I_P$

Similarly, line current, $I_Y = I_{BY} - I_{YR} = \sqrt{3}\, I_P$

And line current, $I_B = I_{RB} - I_{BY} = \sqrt{3}\, I_P$

In a delta network, there is only one phase between any pair of line outers, so the potential difference between the outers, called the line voltage, is equal to phase voltage.

i.e. Line voltage, E_L = phase voltage, E_P

Power output per phase = $E_P I_P \cos\phi$,

where $\cos\phi$ is the power factor of the load.

Total power output, $P = 3E_P I_P \cos\phi$

$= 3E_L \frac{I_L}{\sqrt{3}} \cos\phi$

$= \sqrt{3}\, E_L I_L \cos\phi$

i.e. Total power output = $\sqrt{3}$ x Line voltage x Line current x p.f.

apparent power of 3-phase delta-connected system

= 3 x apparent power per phase

$= 3\, E_P I_P = 3E_L \frac{I_L}{\sqrt{3}} = \sqrt{3}\, E_L I_L$

Each of the two wattmeters connected to measure the input to a three phase reads 20 kW. What does each instrument reads, when the load p.f. is 0.866 lagging with the total three

phase power remaining unchanged in the altered condition?

[6]

sol. : Each wattmeters reads 20 kW Φ $\therefore$ total power = 40 kW $\therefore$ $\Phi = 0$

Now p.f. is 0.866 lag

$\cos\Phi_{new} = 0.866$ $\therefore$ $\Phi_{new} = 30^0$

We have, $\Phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]$ here $\Phi = \Phi_{new}$

As the total power in 3ph circuit remains same $W_1 + W_2 = 410$ --- (i)

$$30^0 = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{40}\right]$$

$$\therefore \frac{(\tan 30^0)40}{\sqrt{3}} = W_1 - W_2$$

$W_1 - W_2 = 13.33$

(i) + (ii) gives,

$\therefore$ $2W_1 = 53.33$

$W_1 = 26.66$ kW

From (i), $W_2 = 40 - W_1 = 40 - 26.66 = 13.33$

$\therefore$ $W_2 = 13.33$ kW

) A balanced star connected load of $(8 + jb)\Omega$ is connected to a 3 phase, 230 V supply.

Find the line current, power factor, power, reactive voltamperes and total voltamperes.

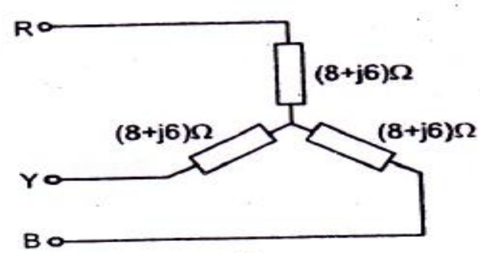


Fig. 9

Sol: $V_L = 230$ volts

$Z_{ph} = 8+j6\Omega = 10\angle 36.86^0\Omega$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.79V$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.79 (0^0}{10 (36.86^0} = 13.27A$

$I_L - I_{ph} = 13.27^0A$

p.f. = $\cos\phi = \cos 36.860 = 0.8lag$

$p = 3V_{ph} I_{ph} \cos\phi = 3x32.79x13.27x0.8 = 4$

Reactive power, $Q = 3V_{ph} I_{ph} \sin\phi = 3x132.79x13.27x0.6$

Q = 3171.82VAR

Total voltamperes, $S = 3V_{ph} \cdot I_{ph} = 3x132.79x13.27$

∴ S = 5286.36

∴

A star connected load consists of 6 Ω resistances in series with an 8 Ω inductive reactance in each phase. A supply voltage of 440 V at 50 Hz is applied to the load. Find the line current, power factor and power consumed by the load. [8]

Sol.: $V_L = 440$ volts, f=50Hz

$R_{Ph} = 6\Omega$

$X_{Ph} = 8\Omega$

Line current = phase current = $\left|\frac{V_{Ph}}{Z_{Ph}}\right|$

$Z_{Ph} = R_{Ph} + jX_{Ph} = 6+ j8 = 10 < 53.13^0\Omega$

$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254$ volts

$I_{Ph} = I_L = \frac{254}{10} = 25.4$ A

Power factor = cos 53.13^0 = 0.6 lagging

$P = \sqrt{3}V_L I_L \cos\phi = \sqrt{3} \times 440 \times 25.4 \times 0.6 = 11.614$ KW

∴ L = 0.3573 H

Each of the two wattmeters connected to measure the input to a three phase reads 20 kW. What does each instrument reads, when the load p.f. is 0.866 lagging with the total three phase power remaining unchanged in the altered condition? [6]

sol. : Each wattmeters reads 20 kW $\therefore$ total power = 40 kW

As both wattmeters reads same cos $\Phi = 1$ $\therefore$ $\Phi = 0^0$

Now p.f. is 0.866 lag

$\text{Cos } \Phi_{new} = 0.866$ $\therefore$ $\Phi_{new} = 30^0$

We have, $\Phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]$ here $\Phi = \Phi_{new}$

As the total power in 3ph circuit remains same $W_1 + W_2 = 410$ ---- (i)

$$30^0 = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{40}\right]$$

$$\therefore \frac{(\tan 30^0)40}{\sqrt{3}} = W_1 - W_2$$

$W_1 - W_2 = 13.33$

(i) + (ii) gives,

$\therefore$ $2W_1 = 53.33$

$W_1 = 26.66$ kW

From (i), $W_2 = 40 - W_1 = 40 - 26.66 = 13.33$

$\therefore$ $W_2 = 13.33$ kW

A 4 pole generator with wave wound armature has 51 slots, each having 24 conductors. The flux per pole is 0.01 Weber. At what speed must the armature rotate to give an induced emf of 220 V? What will be the voltage developed if the voltage developed if the winding is lap and the armature rotates at the same speed? [6]

sol.: P = 4, for wave winding A = 2

Number of slots = 51, conductors/slot = 24

$\varnothing = 0.01$ wb

E = 220 volts

Z = Number of slots × conductors/slots

= 51 × 24

= 1224

$$E = \frac{\varnothing ZNP}{60A}$$

$$\therefore N = \frac{E \times 60A}{\varnothing ZP} = \frac{220 \times 60 \times 2}{0.01 \times 1224 \times 4}$$

$\therefore$ N = 539.21 rpm

Now speed is same but winding is lap $\therefore$ A = P

$$E = \frac{\phi ZNP}{60A} = \frac{\phi ZN}{60} = \frac{0.01 \times 1224 \times 539.21}{60}$$

∴ E = 110 volts

A 4 pole, 220 V. lap connected DC shunt motor has 36 slots, each slot containing 16 conductors. It draws a current of 40 A from the supply. The field resistance and armature resistance are 110Ω, 0.1 Ω respectively. The motor develops an output power of 6 kW. The flux per pole is 40 mwb. Calculate
a) the speed
b) the torque developed by the armature and
c) the shaft torque [7]

Sol.: P = 4, V = 220 volts, slots = 36, A = P = 4
Conductors/slot = 16, Z = 36 × 16 = 576
IL = 40 A
$R_{sh} = 110\Omega$ $R_a = 0.1\Omega$
$P_{output} = 6$ KW, $\phi = 40\times10^{-3}$ wb

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{110} = 2A$$

$I_L = I_a + I_{sh}$

∴ $I_a = I_L - I_{sh}$
= 40 – 2 = 38A

$E_b = V - I_aR_a$

∴ $E_b = 220 - (38)(0.1)$
= 220-3.8

∴ $E_b = 216.2$ volts

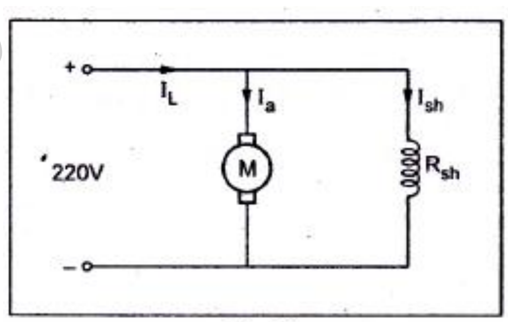


Fig. 9

$$\frac{\phi ZNP}{60A} \qquad \therefore N = \frac{E_b \times 60A}{\phi . Z . P.} = \frac{(216.2)(60)(4)}{(40 \times 10^{-3})(576)(4)}$$

∴ N = 563 rpm

$$T_a = 0.159\ \phi\ Z \frac{I_a P}{A} = 0.159\ \phi\ ZI_a$$

$$= (0.159)(40\times 10^{-3})(576)(38)$$

∴ $T_a = 139.20$ N-m

$$P_{out} = T_{sh} . \frac{2\pi N}{60}$$

$$T_{sh} = \frac{P_{out} \times 60}{2\pi N} = \frac{6 \times 10^3 \times 60}{2\pi \times 563} = 101.76 \text{ N-m}$$

$\therefore$ $T_{sh} = 101.76$ N-m

A 120V DC shunt motor has an armature resistance of 60Ω. It runs at 1800 RPM, when it takes full load current of 40A. Find the speed of the motor while it is operating at half the full load, with load terminal voltage remaining same.

Sol.:

$$I_f = \frac{120}{60} = 2A, \qquad I_{L1} = 40\ A$$

$I_{L1} = I_{a1} + I_f$

$Ia_1 = I_{L1} - I_f = 40 - 2 = 38A$

$E_{b1} = V - I_{a1}R_a = 120 - (38)(0.2)$

$= 120 - 7.6 = 112.4V$

$N_1 = 1800rpm$

Now for shunt machine

$T \propto I_a$ as $\emptyset$ remains constant.

For half load

$$T_2 = \frac{1}{2} T_1$$

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$$

$$\frac{\frac{1}{2}T_1}{T_1} = \frac{I_{a2}}{I_{a1}}$$

$I_{a2} = ½ I_{a1} = ½ (38) = 19A$

$E_{b2} = V - I_{a2}\ Ra = 120 - (19)(0.2) = 120 - 3.8$

$E_{b2} = 116.2V$

Now, $E_b \propto N$ for

Φ constant

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$

$$\frac{116.2}{112.4} = \frac{N_2}{1800}$$

$$N_2 = 116.\frac{2}{112}.4 \times 1800$$

$N_2 = 1860.85$ rpm

RECOMMENDED QUESTIONS

1) Necessity and Advantages of three-phase systems.
2) Explain Generation of 3-phase E.M.F
3) Define phase sequence
4) Obtain Relationship between Line & Phase Values & Expression for power for Balanced Star Connection
5) Obtain Relationship between Line and Phase Values and Expression for Power for Balanced Delta Connection
6) Explain Measurement of power in 3-Phase Circuits Two Wattmeter Method – Balanced Load
7) Explain Measurement of Power Factor – Balanced 3-phase Load

1) What are the advantages of a three phase system over a single phase system?

Sol: Advantages of three phase system:

In the three phase system, the alternator armature has three windings and it produces three independent alternating voltages. The magnitude and frequency of all of them is equal but they have a phase difference of 1200 between each other. Such a three phase system has following advantages over single phase system:

1) The output of three phase machine is always greater than single phase machine of same size, approximately 1.5 times. So for a given size and voltage a three phase alternator occupies less space and has less cost too than single phase having same rating.
2) For a transmission and distribution, three phase system needs less copper or less conducting material than single phase system for given volt amperes and voltage rating so transmission becomes very much economical.
3) It is possible to produce rotating magnetic field with stationary coils by using three phase system. Hence three phase motors are self starting.
4) In single phase system, the instantaneous power is a function of time and hence fluctuates w.r.t. time. This fluctuating power causes considerable vibrations in single phase motors. Hence performance of single phase motors is poor. While instantaneous power in symmetrical three phase system is constant.
5) Three phase system give steady output.
6) Single phase supply can be obtained from three phase but three phase can not be obtained
   From single phase.
7) Power factor of single phase motors is poor than three phase motors of same rating.
8) For converting machines like rectifiers, the d.c. output voltage becomes smoother if number of phases are increased.

But it is found that optimum number of phases required to get all above said advantages is three. Any further increase in number of phases cause a lot of complications. Hence three phase system is accepted as standard polyphase system throughout the world.

2) With a neat circuit diagram and a vector diagram prove that two wattmeters are sufficient to measure total power in a 3 phase system.

.

c) A balanced star connected load of (8 + jb)Ω is connected to a 3 phase, 230 V supply.

Find the line current, power factor, power, reactive voltamperes and total voltamperes.

Sol: $V_L = 230$ volts

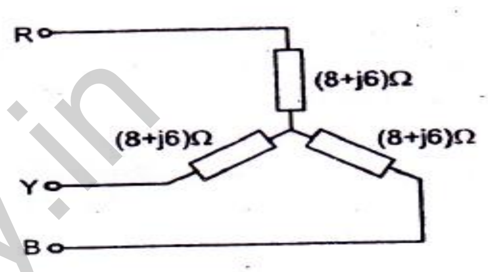


Fig. 9

$Z_{ph} = 8+j6\Omega = 10\angle 36.86^0\Omega$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.79V$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.79\angle 0^0}{10\ \angle 36.86^0} = 13.27A$

$I_L - I_{ph} = 13.27^0 A$

p.f. $= \cos\phi = \cos 36.860 = 0.8$lag

$p = 3V_{ph}\ I_{ph} \cos\phi = 3x32.79x13.27x0.8 = 4$

Reactive power, $Q = 3V_{ph}\ I_{ph} \sin\phi = 3x132.7$

$Q = 3171.82VAR$

Total voltamperes, $S = 3V_{ph} \cdot I_{ph} = 3x132.79x13.27$

$\therefore$ $S = 5286.36$

3)obtain the relationship between line and phase values of current in a three phase balanced, delta connected system.

Sol. : Relation for delta connected load:

Consider the balanced delta connected load as shown in the fig. 10

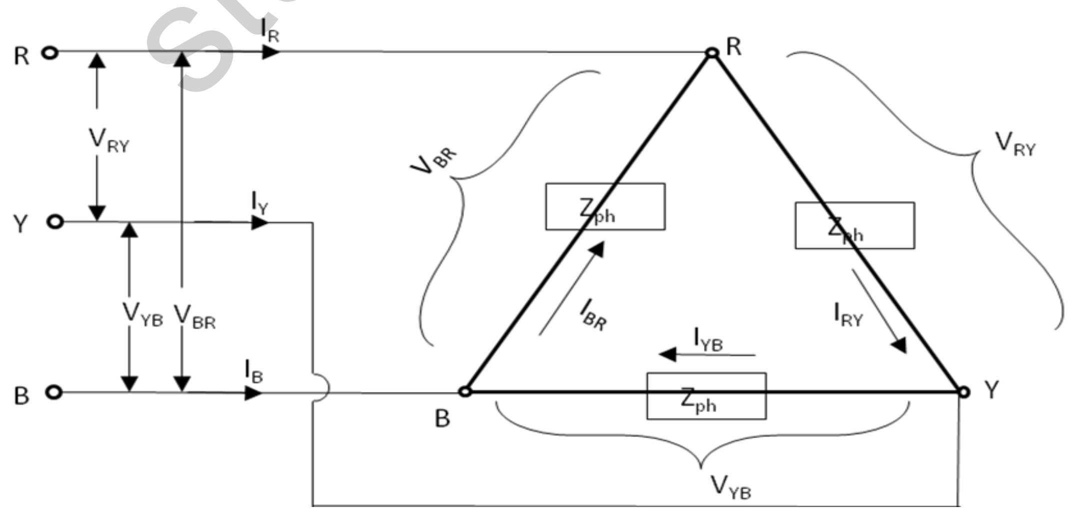


Fig. 10 Delta connected load

Lines voltage $V_L = V_{RY} = V_{YB} = V_{BR}$

Lines currents $I_L = I_R = I_Y = I_B$

Phase voltages $V_{ph} = V_{RY} = V_{YB} = V_{BR}$

Phase currents $I_{Ph} = I_{RY} = I_{YB} = I_{BR}$

As seen earlier, $V_{ph} = V_L$ for delta connected load. To derive the relation between I_L and I_{Ph} apply the KCL at the node R of the load shown in the fig. 10.

$$\Sigma I_{entering} = \Sigma I_{leaving} \text{ at node R}$$

$$\therefore \quad \bar{I}_R + \bar{I}_{BR} = \bar{I}_{RY}$$

$$\therefore \quad \bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}$$

Apply KCL at node Y and B we can write equations for line currents I_Y and I_B as,

The phasor diagram to obtain line Current I_R by carrying out vector subtraction Of phase currents I_{RY} and I_{YB} is shown in the fig. 11.

The perpendicular AC drawn on vector OB, bisects the vector OB which represents I_L.

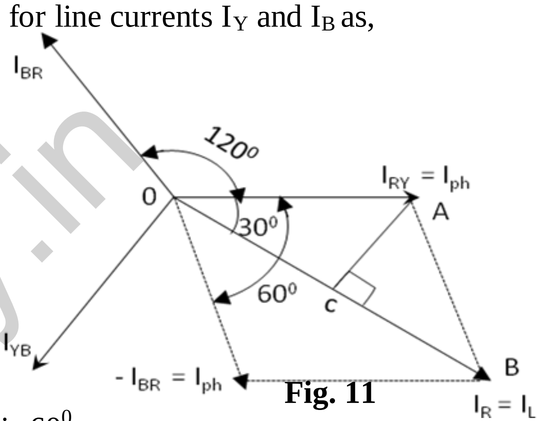


Fig. 11

similarly OB bisects angle between - I_{YB} and I_{RY} which is 60^0

$\therefore \angle BOA = 30^0$

And $OC = CB = \frac{I_L}{2}$

From triangle OAB, $\cos 30^0 = \frac{OC}{OA} = \frac{I_R/2}{I_{RY}}$

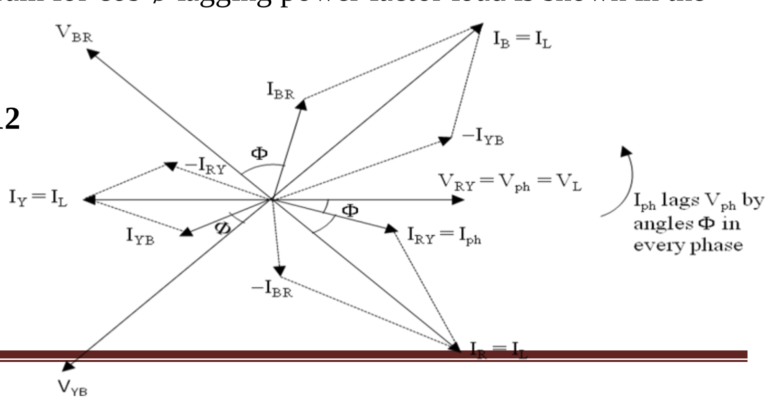
$$\therefore \quad I_L \frac{\sqrt{3}}{2} = \frac{I_L/2}{I_{ph}}$$

$$\therefore \quad \sqrt{3}\, I_{ph} \quad \text{for delta connection}$$

Again Z_{ph} decides wheather I_{ph} has to lag, lead to remain in phase with V_{ph}. Angle between V_{ph} and I_{ph} is $\emptyset$.

The complete phasor diagram for cos $\emptyset$ lagging power factor load is shown in the fig. 12.

Fig. 12



$$Z_{ph} = R_{ph} + jX_{Lph} = |Z_{ph}| \angle \Phi \Omega$$

Each I_{ph} lags respective V_{ph} by the angle Φ

Power: Power consumed in each phase is single phase power given by,

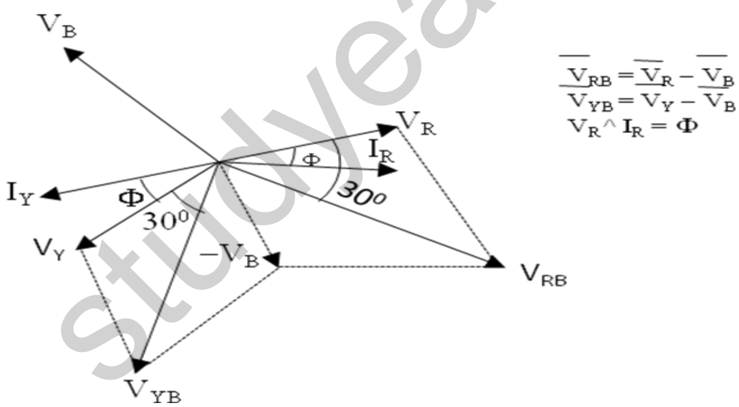
$$P_{ph} = V_{ph} I_{ph} \cos\emptyset$$

Total power $p = 3 P_{ph} = 3 V_{ph} I_{ph} \cos\emptyset = 3V_L \frac{I_L}{\sqrt{3}} \cos\emptyset$

$$P = \sqrt{3} V_L I_L \cos\emptyset$$

4) Show that in a three phase, balanced circuit, two wattmeters are sufficient to measure the total three phase power and power factor of the circuit.

Sol. : Two wattmeter method: The current coils of the two wattmeters are connected in any two lines while the voltage coil of each wattmeters is connected between its own current coil terminal and line without current coil. Consider star connected balanced load and two wattmeters connected as shown in fig. 13. Let us consider the rms values of the currents and voltages to prove that sum of two wattmeter gives total power consumed by three phase load.



$$V_{RB} = V_R - V_B$$

$$V_{YB} = V_Y - V_B$$

$$V_R \wedge I_R = \Phi$$

$$V_R = V_Y = V_B = V_{ph}$$

$$V_{RB} = V_R - V_B, \quad I_R = I_Y = I_L = I_{ph}$$

$$V_{YB} = V_Y - V_B = V_{RB} = V_L$$

From fig. 14, $I_R \wedge V_{RB} = 30 - \Phi$ and $I_R \wedge V_{RB} = 30 + \Phi$

$$W_1 = I_R V_{RB} \cos(30 - \Phi) = V_L I_L \cos(30 - \Phi)$$

$$W_2 = I_Y V_{YB} \cos(30 + \Phi) = V_L I_L \cos(30 + \Phi)$$

$$W_1 + W_2 = V_L I_L [\cos(30 - \Phi) + \cos(30 + \Phi)]$$

$= V_L I_L[\cos 30 \cos\Phi + \sin 30 \sin \Phi + \cos 30 \cos \Phi - \sin 30 \sin \Phi]$

$= 2V_L I_L \cos 30 \cos \Phi = 2V_L I_L \frac{\sqrt{3}}{2} \cos \Phi$

$= \sqrt{3}\, V_L I_L \cos \Phi$

= total power

5) Each of the two wattmeters connected to measure the input to a three phase reads 20 kW. What does each instrument reads, when the load p.f. is 0.866 lagging with the total three phase power remaining unchanged in the altered condition? [6]

sol. : Each wattmeters reads 20 kW $\therefore$ total power = 40 kW

As both wattmeters reads same cos Φ $\therefore$ $\Phi = 0^0$

Now p.f. is 0.866 lag

$\text{Cos } \Phi_{new} = 0.866$ $\therefore$ $\Phi_{new} = 30^0$

We have, $\Phi = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}\right]$ here $\Phi = \Phi_{new}$

As the total power in 3ph circuit remains same $W_1 + W_2 = 410$ ------- (i)

$$30^0 = \tan^{-1}\left[\frac{\sqrt{3}(W_1 - W_2)}{40}\right]$$

$$\therefore \frac{(\tan 30^0)40}{\sqrt{3}} = W_1 - W_2$$

$W_1 - W_2 = 13.33$

(i) + (ii) gives,

$\therefore$ $2W_1 = 53.33$

$W_1 = 26.66$ kW

From (i), $W_2 = 40 - W_1 = 40 - 26.66 = 13.33$

$\therefore$ $W_2 = 13.33$ kW

6) Three similar coils each having resistance of 10 ohm and reactance of 8 ohm are connected in star across a 400 V, 3 phase supply . Determine the i) Line current; ii) Total power and iii) Reading of each of two wattmeters connected to measure the power. **(8)**

Ans.: $R=10\ \Omega$, $X_L= 8\ \Omega$, $V_L= 400$ V, star

$\therefore$ $Z_{ph} = 10 + j\,8\ \Omega = 12.082 \angle 38.659^0 \Omega$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94$ V,

…star

i) $I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{12.8062} = 18.0334$ V,

∴ $I_L = I_{ph} = 18.0334$ A.

ii) $P_T = \sqrt{3}\ V_L\, I_L \cos\Phi$ where $\Phi = 38.659^0$

$= \sqrt{3} \times 400 \times 18.0334 \times \cos 38.659^0$

$= 9756.2116$ W

iii) $W_1 = V_L I_L \cos(30 - \Phi) = 400 \times 18.0334 \times \cos(30^0 - 38.659^0)$

= 7131.1412 W

$W_2 = V_L I_L \cos(30 + \Phi) = 400 \times 18.0334 \times \cos(30^0 + 38.659^0)$

= 2625.0704 W

7) A balanced star connected load of (8 + jb) Ω is connected to a 3 phase, 230 V supply. Find the line current, power factor, power, reactive voltamperes and total voltamperes.

Sol: $V_L = 230$ volts

$Z_{ph} = 8 + j6\Omega = 10\angle 36.86^0\Omega$

$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.79V$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.79\angle 0^0}{10\ \angle 36.86^0} = 13.27A$

$I_L - I_{ph} = 13.27^0 A$

p.f. = cos ϕ = cos36.860 = 0.8lag

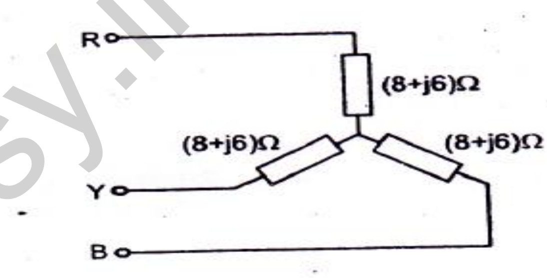


Fig. 9

8) obtain the relationship between line and phase balanced, delta connected system.

Sol. : Relation for delta connected load:

Consider the balanced delta connected load as shown in the fig. 10

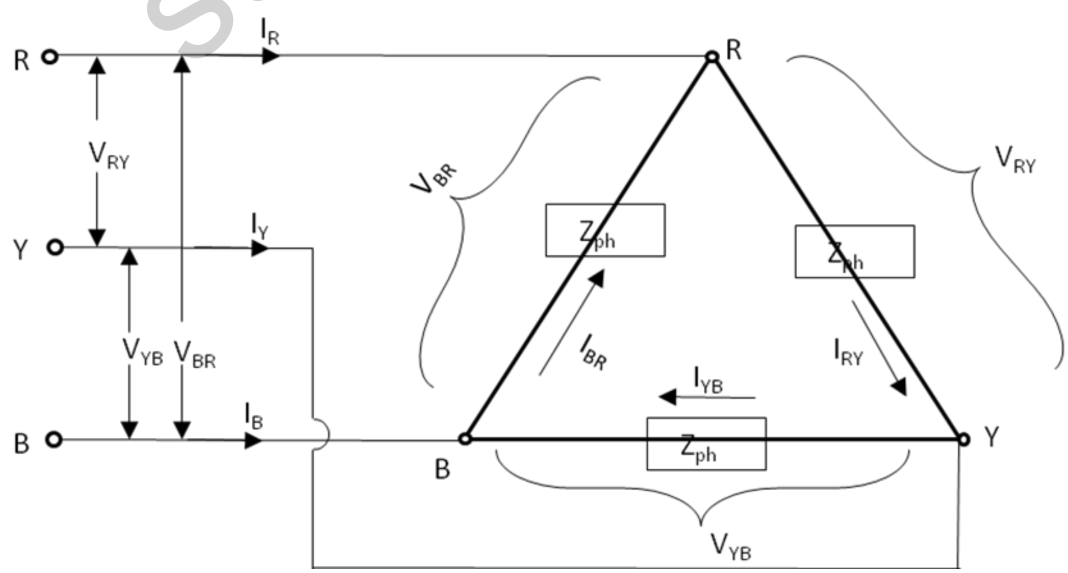


Fig. 10 Delta connected load

Lines voltage $V_L = V_{RY} = V_{YB} = V_{BR}$

Lines currents $I_L = I_R = I_Y = I_B$
Phase voltages $V_{ph} = V_{RY} = V_{YB} = V_{BR}$
Phase currents $I_{Ph} = I_{RY} = I_{YB} = I_{BR}$

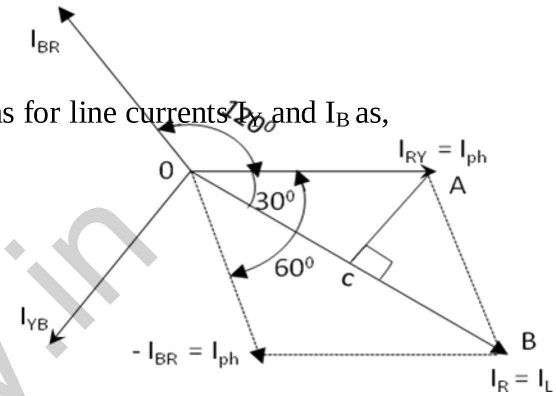
As seen earlier, $V_{ph} = V_L$ for delta connected load. To derive the relation between I_L and I_{Ph} apply the KCL at the node R of the load shown in the fig. 10.

$$\Sigma I_{entering} = \Sigma I_{leaving} \text{ at node R}$$

$$\therefore \quad \bar{I}_R + \bar{I}_{BR} = \bar{I}_{RY}$$

$$\therefore \quad \bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}$$

Apply KCL at node Y and B we can write equations for line currents I_Y and I_B as,



The phasor diagram to obtain line Current I_R by carrying out vector subtraction Of phase currents I_{RY} and I_{YB} is shown in the fig. 11.

The perpendicular AC drawn on vector OB, bisects the vector OB which represents I_L.

Fig. 11

similarly OB bisects angle between - I_{YB} and I_{RY} which is 60^0

UNIT-4

Measuring Instruments: Construction and Principle of operation of dynamometer type wattmeter and single-phase induction type energy meter (problems excluded).
03 Hours

4–b) Domestic Wiring. Brief discussion on Cleat, Casing & Capping and conduit (concealed) wiring. Two-way and three-way control of a lamp. Elementary discussion on fuse and Miniature Circuit Breaker (MCB's). Electric shock, precautions against shock – Earthing: Pipe and Plate

UNIT-4

DOMESTIC WIRING

Introduction

A network of wires drawn connecting the meter board to the various energy consuming loads (lamps, fans, motors etc) through control and protective devices for efficient distribution of power is known as electrical wiring.
Electrical wiring done in residential and commercial buildings to provide power for lights, fans, pumps and other domestic appliances is known as domestic wiring. There are several wiring systems in practice. They can be classified into:

Types of wiring: Depending upon the above factors various types of wiring used in practice are:

1. Cleat wiring
2. Casing wiring
3. Surface wiring
4. Conduit wiring

i) Clear wiring:

In this type V.I.R or P.V.C wires are clamped between porcelain cleats.

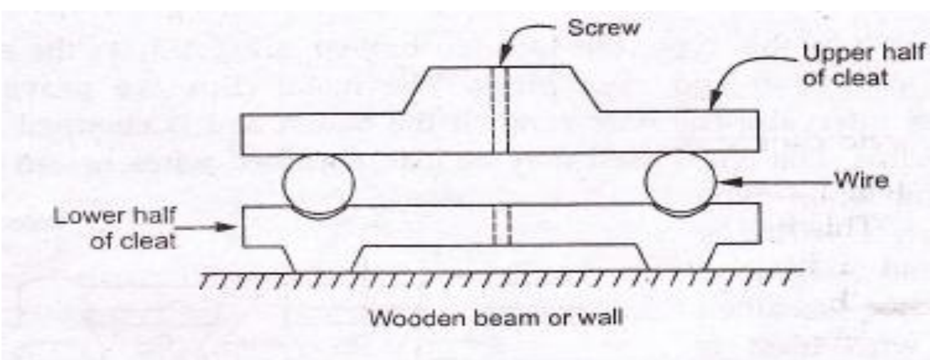


Fig. 16 Cleat wiring

The cleats are made up of two halves. One half is grooved through which wire passes while the other fits over the first. The whole assembly is then mounted on the wall or wooden beam with the help of screws.

This method is one of the cheapest method and most suitable for temporary work. It can be very quickly installed and can be recovered without any damage of material. Inspection and changes can be made very easily.

This method does not give attractive appearance. After some time due to sagging at some places, it looks shabby. Dust and dirt collects on the cleats. The wires are directly exposed to atmospheric conditions like moisture, chemical fumes etc. maintenance cost is very high.

Due to these disadvantages this type is not suitable for permanent jobs.

ii) Casing capping: This is very popularly used for residential buildings. In this method, casing is a rectangular strip made from teak wood or new a day's made up of P.V.C. It has two grooves into which the wires are laid. Then casing is covered with a rectangular strip of wood or P.V.C. of the same width, called capping. The capping is screwed into casing is fixed to the walls the help or porcelain discs or cleats.

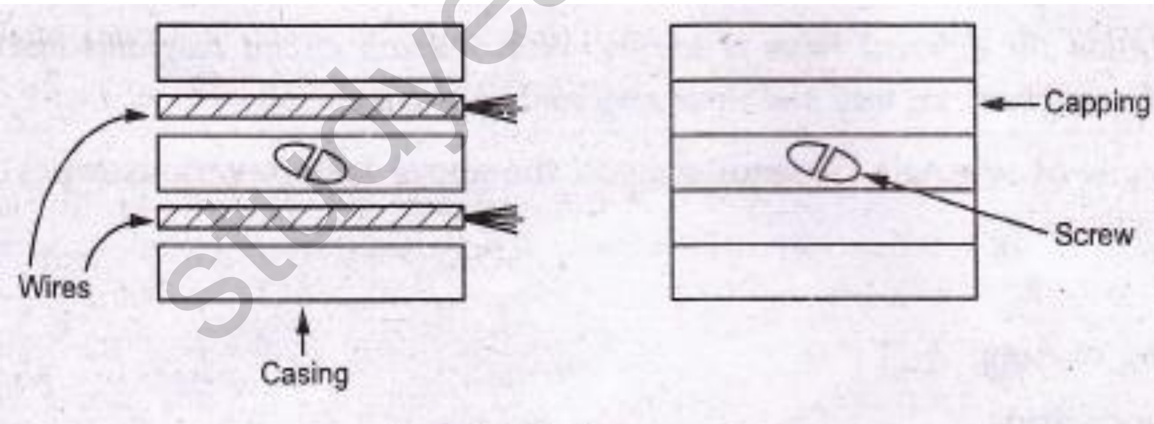


Fig. 17 Casing capping

Good protection to the conductors from dangerous atmospheric conditions, neat and clean appearance are the advantages of this type.

In case of wooden casing capping, there is high risk of fire along with the requirement of skilled labour. The method is costly.

Surface wiring: in this type, the wooden battens are fixed on the surface of the wall, by means of screws and rawl plugs. The metal clips are provided with the battens at regular intervals. The wire runs on the batten and is clamped on the batten using the metal clips. The wires used may lead sheathed wires or can tyre sheathed wires.

Depending upon type of wire used surface wiring is also called lead Sheathed wiring or cab tyre sheathed wiring. If the wire used is though rubber Sheathed then it is called T.R.S. wiringWhile if the wire used is cab tyre SheathedThen it is called C.T.S wiring.

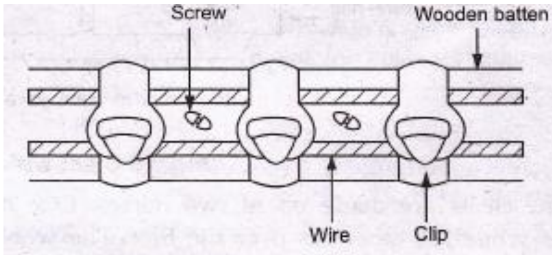


Fig. 18 Wooden batten wiring

Conduit wiring: In this method, metallic tubes called as conduits are used to run the wires. This is the best system of wiring as it gives full mechanical protection to the wires. This is most desirable for workshops and public Buildings. Depending on whether the conduits are laid inside the walls or supported on the walls, there are two types of conduit wiring which are :

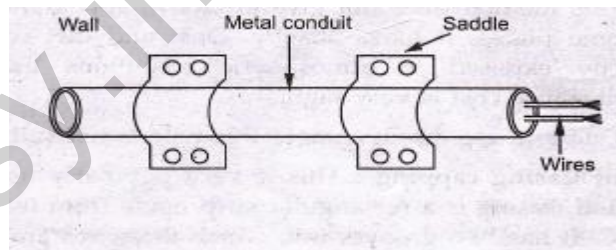


Fig. 19 Surface conduit wiring

i) **Surface conduit wiring:** in this method conduits are mounted or supported on the walls with the help of pipe books or saddles. In damp situations, the conduits are spaced apart from the wall by means of wooden blocks.

ii) **Concealed conduit wiring:** In this method, the conduit are buried under the wall aty the some of plastering. This is also called recessed conduit wiring.

The beauty of the premises is maintained due to conduit wiring. It is durable and has long life. It protects the wires from mechanical shocks and fire hazards. Proper earthing of conduits makes the method electrical shock proof. It requires very less maintenance.

The repairs are very difficult in case of concealed conduit wiring. This method is most costly and erection requires highly skilled labour. These are few disadvantages of the conduit type of wiring. In concealed conduit wiring, keeping conduit at earth potential is must.

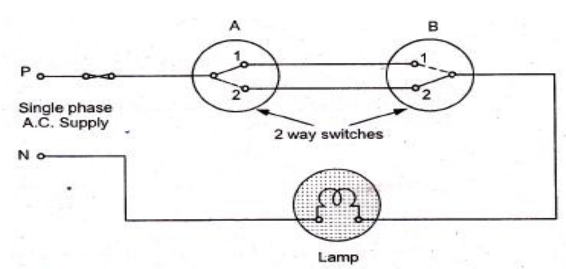


Fig. 20 Control of one from two points

FACTORS AFFECTING THE CHOICE OF WIRING SYSTEM:

The choice of wiring system for a particular installation depends on technical factors and economic viability.

1. Durability: Type of wiring selected should conform to standard specifications, so that it is durable i.e. without being affected by the weather conditions, fumes etc.
2. Safety: The wiring must provide safety against leakage, shock and fire hazards for the operating personnel.
3.Appearance: Electrical wiring should give an aesthetic appeal to the interiors.
4. Cost: It should not be prohibitively expensive.
5. Accessibility: The switches and plug points provided should be easily accessible. There must be provision for further extension of the wiring system, if necessary.
6 Maintenance Cost: The maintenance cost should be a minimum
7. Mechanical safety: The wiring must be protected against any mechanical damage

Specification of Wires:

The conductor material, insulation, size and the number of cores, specifies the electrical wires. These are important parameters as they determine the current and voltage handling capability of the wires. The conductors are usually of either copper or aluminum. Various insulating materials like PVC, TRS, and VIR are used. The wires may be of single strand or multi strand. Wires with combination of different diameters and the number of cores or strands are available.

For example: The VIR conductors are specified as 1/20, 3/22,….7/20 ………

The numerator indicates the number of strands while the denominator corresponds to the diameter of the wire in SWG (Standard Wire Gauge). SWG 20 corresponds to a wire of diameter 0.914mm, while SWG 22 corresponds to a wire of diameter 0.737 mm.

A 7/0 wire means, it is a 7-cored wire of diameter 12.7mm (0.5 inch). The selection of the wire is made depending on the requirement considering factors like current and voltage ratings, cost and application.

Example: Application: domestic wiring

1. Lighting - 3/20 copper wire
2. Heating - 7/20 copper wire

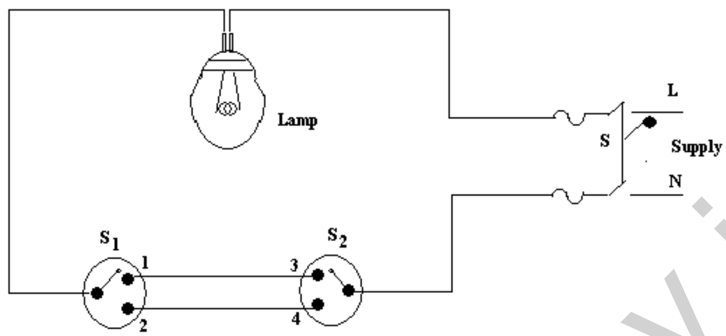
The enamel coating (on the individual strands) mutually insulates the strands and the wire on the whole is provided with PVC insulation. The current carrying capacity depends on the total area of the wire. If cost is the criteria then aluminum conductors are preferred. In that case, for the same current rating much larger diameter of wire is to be used.

Two- way and Three- way Control of Lamps:

The domestic lighting circuits are quite simple and they are usually controlled from one point. But in certain cases it might be necessary to control a single lamp from more than one point (Two or Three different points).
For example: staircases, long corridors, large halls etc.

<u>Two-way Control of lamp</u>:

Two-way control is usually used for staircase lighting. The lamp can be controlled from two different points: one at the top and the other at the bottom - using two- way switches which strap wires interconnect. They are also used in bedrooms, big halls and large corridors. The circuit is shown in the following figure.



Two -way control of lamp

Switches $\mathbf{S_1}$ and $\mathbf{S_2}$ are two-way switches with a pair of terminals 1&2, and 3&4 respectively. When the switch $\mathbf{S_1}$ is in position**1** and switch $\mathbf{S_2}$ is in position **4**, the circuit does not form a closed loop and there is no path for the current to flow and hence the lamp will be **OFF.** When $\mathbf{S_1}$ is changed to position **2** the circuit gets completed and hence the lamp glows or is **ON**. Now if $\mathbf{S_2}$ is changed to position **3** with $\mathbf{S_1}$ at position **2** the circuit continuity is broken and the lamp is off. Thus the lamp can be controlled from two different points.

Position of S1	Position of S2	Condition of lamp
1	3	ON
1	4	OFF
2	3	OFF
2	4	ON

<u>Three- way Control of lamp:</u>

In case of very long corridors it may be necessary to control the lamp from 3 different points. In such cases, the circuit connection requires two; two-way switches $\mathbf{S_1}$and $\mathbf{S_2}$ and an intermediate switch $\mathbf{S_3}$. An intermediate switch is a combination of two, two way switches coupled together. It has 4 terminals ABCD. It can be connected in two ways

a) Straight connection
b) Cross connection

In case of straight connection, the terminals or points AB and CD are connected as shown in figure 1(a) while in case of cross connection, the terminals AB and C D is connected as shown in figure 1(b). As explained in two –way control the lamp is ON if the circuit is complete and is OFF if the circuit does not form a closed loop.

Three -way control of lamp

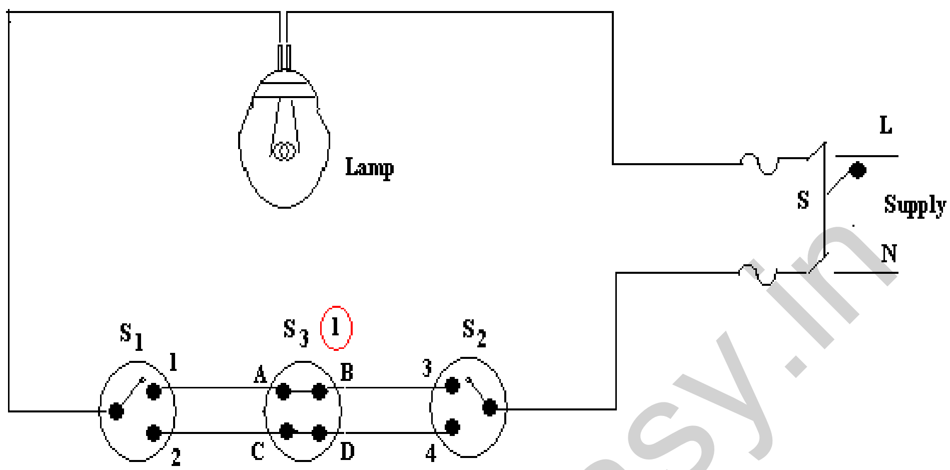


Figure 1 (a) Straight connection

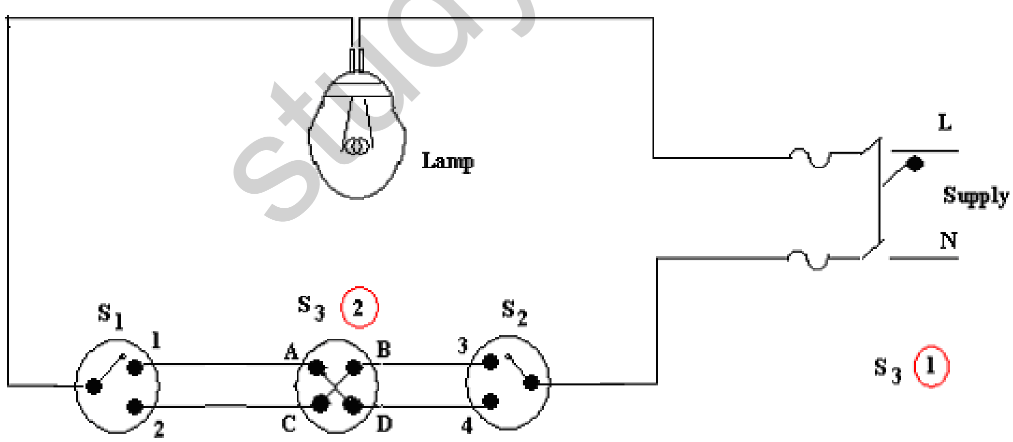


Figure 1 (b) Cross connection

The condition of the lamp is given in the table depending on the positions of the switches **S_1**, **S_2** and **S_3**.

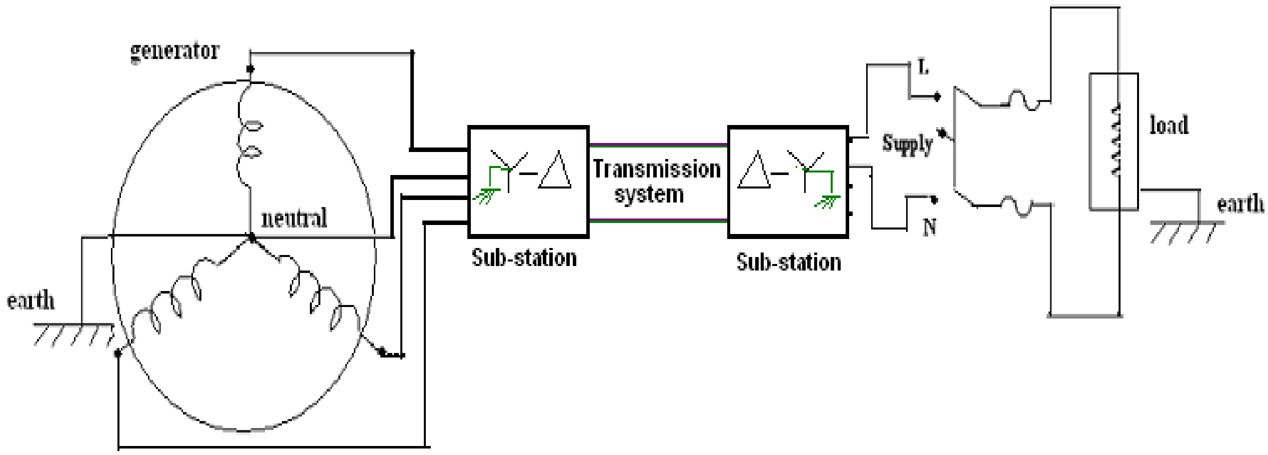
p

Position of S3	**Position of S1**	**Position of S2** **lamp** **lamp**	**Condition of the**
	1	3	**ON**
1	1	4	**OFF**
Straight	2	3	**OFF**
connection	2	4	**ON**
	1	3	**OFF**
2	1	4	**ON**
Cross	2	3	**ON**
connection	2	4	**OFF**

EARTHING:

The potential of the earth is considered to be at zero for all practical purposes as the generator (supply) neutral is always earthed. The body of any electrical equipment is connected to the earth by means of a wire of negligible resistance to safely discharge electric energy, which may be due to failure of the insulation, line coming in contact with the casing etc. Earthing brings the potential of the body of the equipment to ZERO i.e. to the earth's potential, thus protecting the operating personnel against electrical shock. The body of the electrical equipment is not connected to the supply neutral because due to long transmission lines and intermediate substations, the same neutral wire of the generator will not be available at the load end. Even if the same neutral wire is running it will have a self-resistance, which is higher than the human body resistance. Hence, the body of the electrical equipment is connected to earth only.

Thus earthing is to connect any electrical equipment to earth with a very low resistance wire, making it to attain earth's potential. The wire is usually connected to a copper plate placed at a depth of 2.5 to 3meters from the ground level.

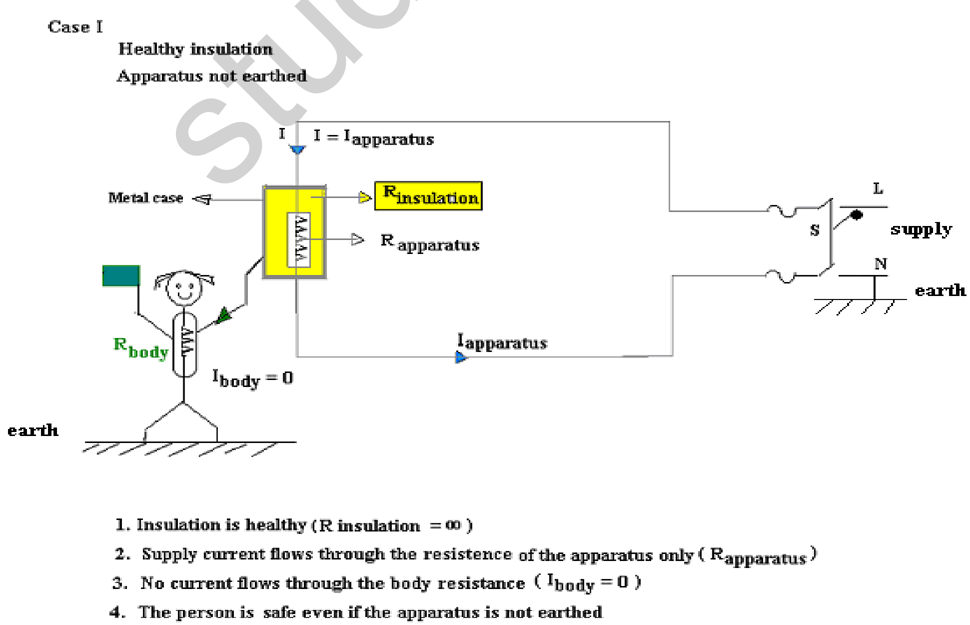


BLOCK DIAGRAM

The earth resistance is affected by the following factors:

1. Material properties of the earth wire and the electrode
2. Temperature and moisture content of the soil
3. Depth of the pit
4. Quantity of the charcoal used

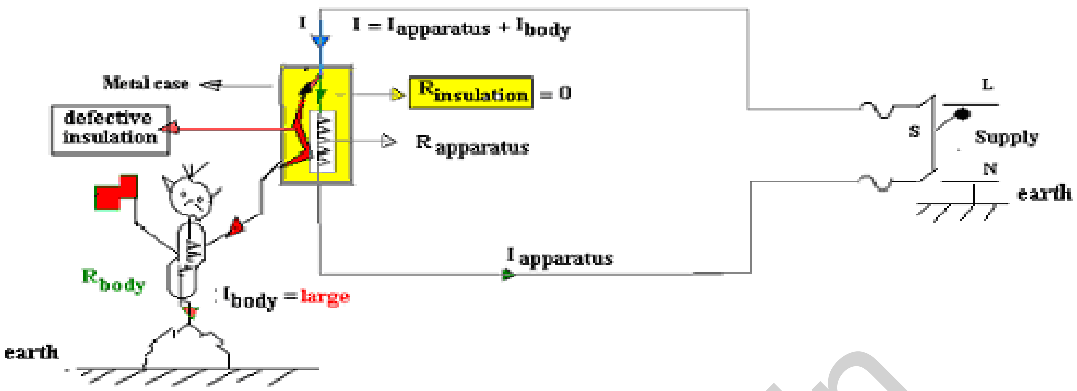
The importance of earthing is illustrated in the following figures



Case II

Defective insulation

Apparatus not earthed

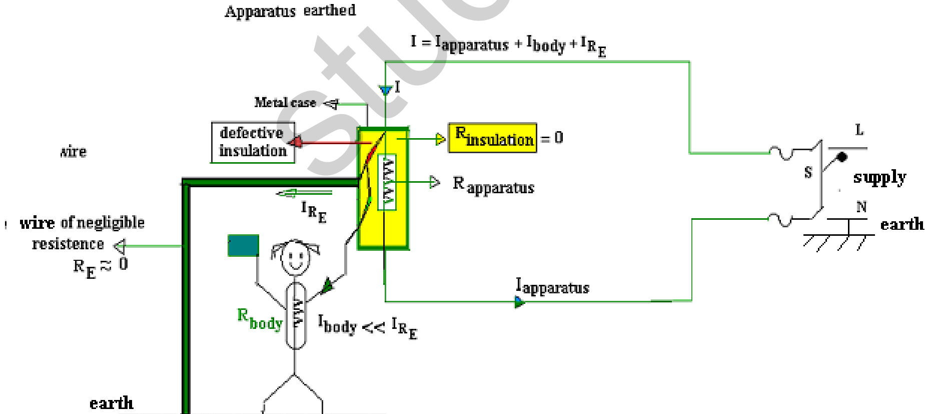


1. Insulation is bad (R insulation = 0)
2. Supply current now divides into $I_{apparatus}$ and I_{body}
3. A part of the supply current flows through the body to the ground I_{body}
4. The person experiences shock as the apparatus is not earthed

Case III

Defective insulation

Apparatus earthed



1. Insulation is bad (R insulation = 0)
2. Supply current now divides into $I_{apparatus}$ I_{body} and I_{R_E}
3. A part of the supply current I_{body} flows through the body to the ground
4. Now I_{body} is very less compared to the current flowing through wire of negligible resistance connecting the apparatus metal case to ground $I_{body} << I_{R_E}$
5. The person in contact with the apparatus does not experience any shock as the metal casing is earthed

❖ **Necessity of Earthing**:

1. To protect the operating personnel from danger of shock in case they come in contact with the charged frame due to defective insulation.
2. To maintain the line voltage constant under unbalanced load condition.
3. Protection of the equipments
4. Protection of large buildings and all machines fed from overhead lines against lightning.

❖ **Methods of Earthing**:

The important methods of earthing are the plate earthing and the pipe earthing. The earth resistance for copper wire is 1 ohm and that of G I wire less than 3 ohms. The earth resistance should be kept as low as possible so that the neutral of any electrical system, which is earthed, is maintained almost at the earth potential. The typical value of the earth resistance at powerhouse is 0. 5 ohm and that at substation is 1 ohm.

1. Plate earthing 2. Pipe earthing

Plate Earthing

In this method a copper plate of 60cm x 60cm x 3.18cm or a GI plate of the size 60cm x 60cm x 6.35cm is used for earthing. The plate is placed vertically down inside the ground at a depth of 3m and is embedded in alternate layers of coal and salt for a thickness of 15 cm. In addition, water is poured for keeping the earth electrode resistance value well below a maximum of 5 ohms. The earth wire is securely bolted to the earth plate. A cement masonry chamber is built with a cast iron cover for easy regular maintenance.

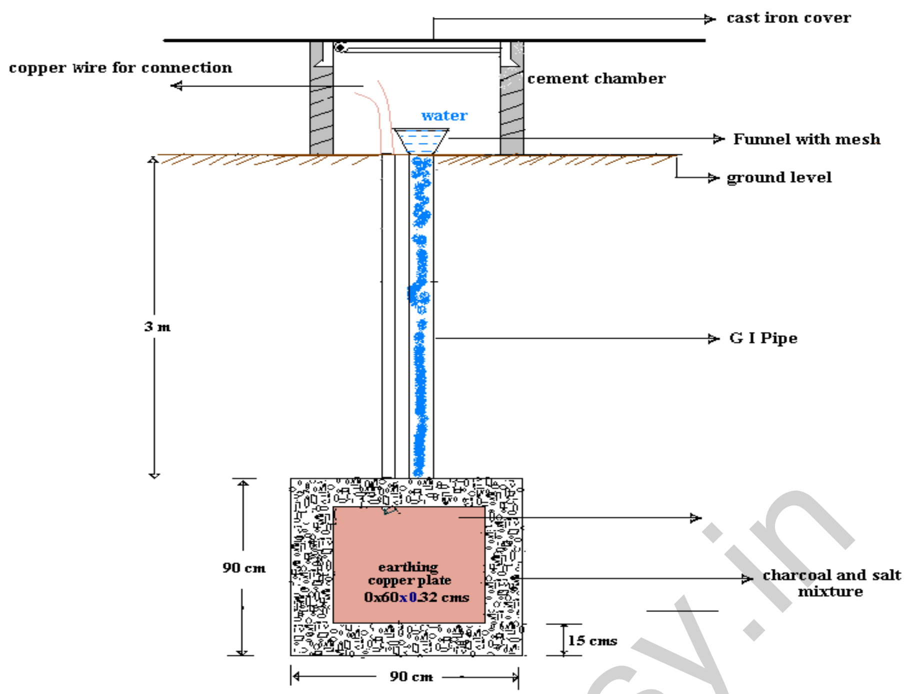
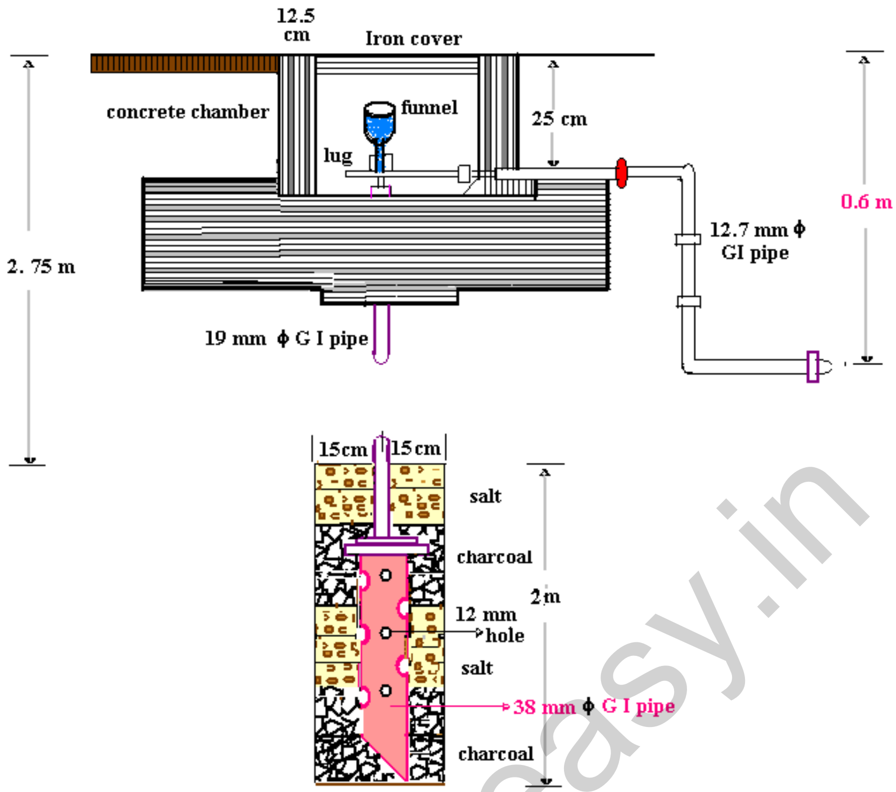


PLATE EARTHING

Pipe Earthing

Earth electrode made of a GI (galvanized) iron pipe of 38mm in diameter and length of 2m (depending on the current) with 12mm holes on the surface is placed upright at a depth of 4.75m in a permanently wet ground. To keep the value of the earth resistance at the desired level, the area (15 cms) surrounding the GI pipe is filled with a mixture of salt and coal.. The efficiency of the earthing system is improved by pouring water through the funnel periodically. The GI earth wires of sufficient cross- sectional area are run through a 12.7mm diameter pipe (at 60cms below) from the 19mm diameter pipe and secured tightly at the top as shown in the following figure.



PIPE EARTHING

When compared to the plate earth system the pipe earth system can carry larger leakage currents as a much larger surface area is in contact with the soil for a given electrode size. The system also enables easy maintenance as the earth wire connection is housed at the ground level.

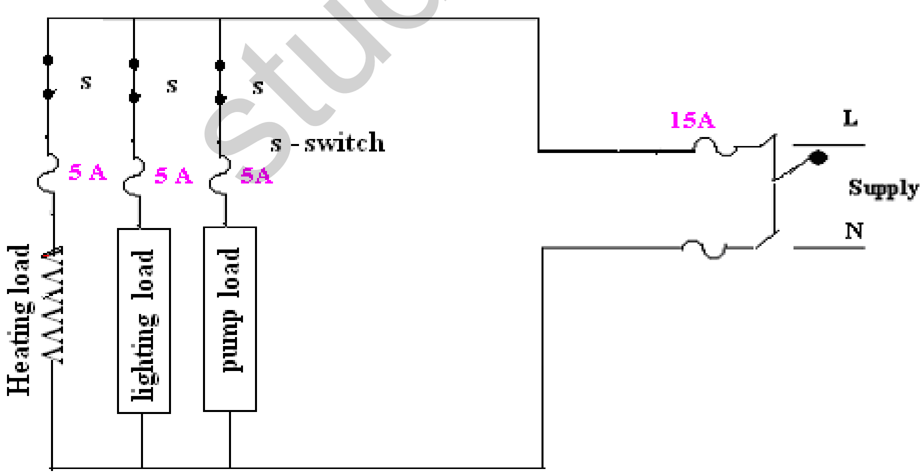
PROTECTIVE DEVICES

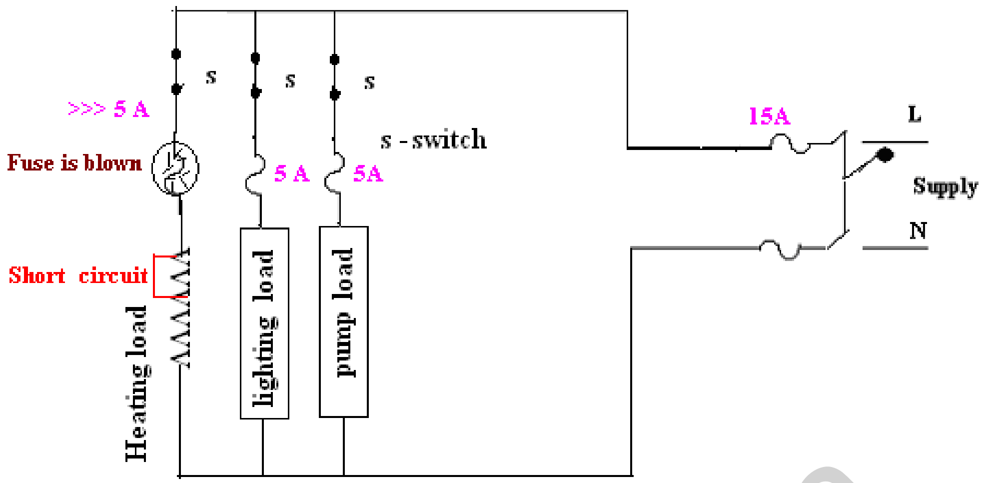
Protection for electrical installation must be provided in the event of faults such as short circuit, overload and earth faults. The protective circuit or device must be fast acting and isolate the faulty part of the circuit immediately. It also helps in isolating only required part of the circuit without affecting the remaining circuit during maintenance. The following devices are usually used to provide the necessary protection:

- Fuses
- Relays
- Miniature circuit breakers (MCB)
- Earth leakage circuit breakers (ELCB)

FUSE

The electrical equipments are designed to carry a particular rated value of current under normal circumstances. Under abnormal conditions such as short circuit, overload or any fault the current raises above this value, damaging the equipment and sometimes resulting in fire hazard. Fuses are pressed into operation under such situations. Fuse is a safety device used in any electrical installation, which forms the weakest link between the supply and the load. It is a short length of wire made of lead / tin /alloy of lead and tin/ zinc having a low melting point and low ohmic losses. Under normal operating conditions it is designed to carry the full load current. If the current increases beyond this designed value due any of the reasons mentioned above, the fuse melts (said to be blown) isolating the power supply from the load as shown in the following figures.





▪ CHARACTERISTICS OF FUSE MATERIAL

The material used for fuse wires must have the following characteristics

1. Low melting point
2. Low ohmic losses
3. High conductivity
4. Lower rate of deterioration

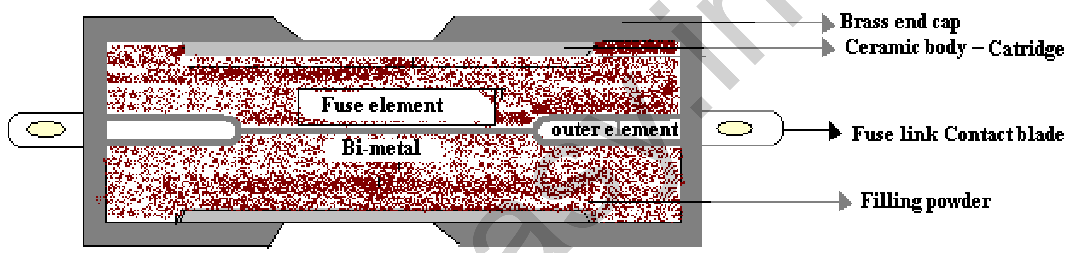
▪ Different types of fuses:

- **Re-wirable or kit -kat fuses:** These fuses are simple in construction, cheap and available up-to a current rating of 200A. They are erratic in operation and their performance deteriorates with time.
- **Plug fuse**: The fuse carrier is provided with a glass window for visual inspection of the fuse wire.
- **Cartridge fuse**: Fuse wire usually an alloy of lead is enclosed in a strong fiber casing. The fuse element is fastened to copper caps at the ends of the casing. They are available up-to a voltage rating of 25kV. They are used for protection in lighting installations and power lines.
- **Miniature Cartridge fuses**: These are the miniature version of the higher rating cartridge fuses, which are extensively used in automobiles, TV sets, and other electronic equipments.
- **Transformer fuse blocks:** These porcelain housed fuses are placed on secondary of the distribution transformers for protection against short circuits and overloads.
- **Expulsion fuses:** These consist of fuse wire placed in hollow tube of fiber lined with asbestos. These are suited only for out door use for example, protection of high voltage circuits.
- **Semi-enclosed re-wirable fuses:** These have limited use because of low breaking capacity.

- **Time-delay fuse:** These are specially designed to withstand a current overload for a limited time and find application in motor circuits.

HRC CARTRIDGE FUSE

The high rupturing capacity or (HRC) fuse consists of a heat resistant ceramic body. Then silver or bimetallic fuse element is welded to the end brass caps. The space surrounding the fuse element is filled with quartz powder. This filler material absorbs the arc energy and extinguishes it.
When the current exceeds the rated value the element melts and vaporizes. The vaporized silver fuses with the quartz and offers a high resistance and the arc is extinguished.



H R C Catridge fuse

Advantages:
1. Fast acting
2. Highly reliable
3. Relatively cheaper in comparison to other high current interrupting device

Disadvantages:
2. Requires replacement
3. The associated high temperature rise will affect the performance of other devices

TERMS RELATED WITH FUSES

Rated current: It is the maximum current, which a fuse can carry without undue heating or melting. It depends on the following factors:
1. Permissible temperature rise of the contacts of the fuse holder and the fuse material
2. Degree of deterioration due to oxidation

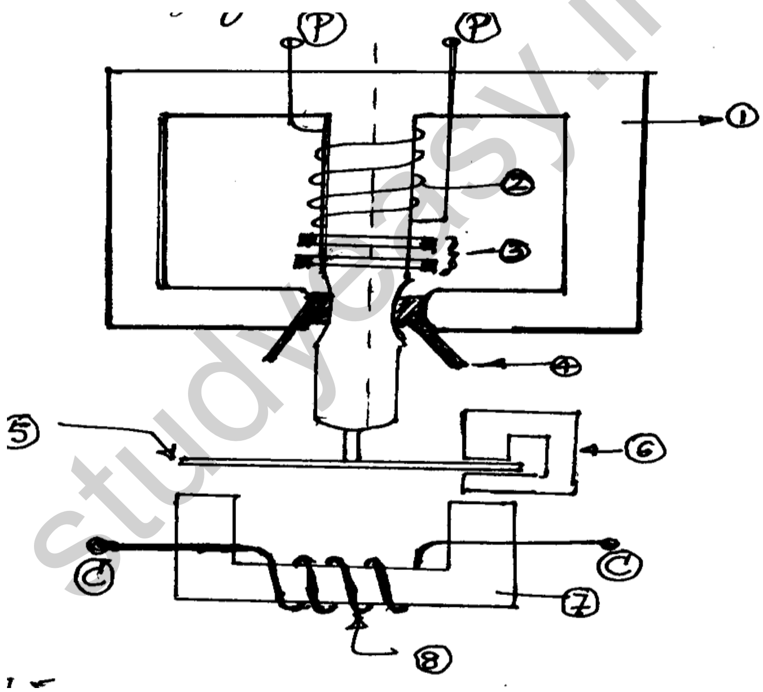
Fusing current: The minimum current at which the fuse melts is known as the fusing current. It depends on the material characteristics, length, diameter, cross-sectional area of the fuse element and the type of enclosure used.

Fusing Factor: It is the ratio of the minimum fusing current to the rated current. It is always greater than unity.

ELECTRICAL MEASURING INSTRUMENTS

INDUCTION TYPE ENERGY METER:-

This is a measuring instrument/device which works on the principle of induction and measures the energy consumed over a definite period.



1) Upper Magnet/ shunt magnet (P.P)
2) Potential coil/ Voltage coil
3) Copper Shading bands
4) Friction compensator
5) Aluminium disc
6) Brake magnet
7) Lower magnet/Series magnet
8) Current coil (C-C)

This instrument consisting two electromagnets as in fig.

1. **Upper magnet or Shunt magnet**: which carries the potential coil on its central limb which also carries one or two copper shading bands for the power factor adjustment.
2. **Lower magnet or Series magnet:** Which carries the current coil as shown.

An aluminum disc is between the fields of the upper and lower electro magnets. There is a friction compensator in the upper magnets for the measurement at very low loads. The aluminum disc rotates in the field of a brake magnet whose position can be set so that the disc rotates at proper speeds at higher loads.

This instrument works on the principle of induction that when both the shunt and series coils are energized by ac, there will be tow alternative fluxes are in the shunt coil and one in the series coil these time varying fluxes are cut by a stationary disc. Inducing currents in the disc. These currents interacts with the fluxes and results in a torque which is given by

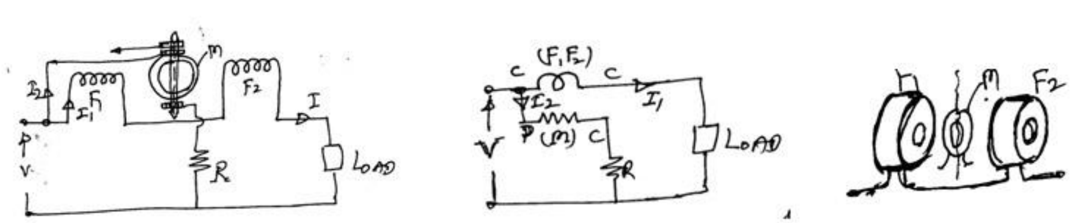
$T\alpha (K_1\varphi_{sh} i_{se} + K_2\varphi_{se} i_{sh})$ there by the disc rotates in a particular direction and the number and speed of rotations depends on the energy consumed by the load.

Some times the energy meters disc rotates slowly even on no load conditions as the potential coil is continuously energized and this effect is called the 'CREEP' and the speed is called the 'CREEP SPEED' to minimum this creep one pair of diametrically opposite holes are made in the aluminum disc which alters the reluctance and minimizes the creep effect.

DYNAMOMETER WATTMETER:-

In this type there will not be any permanent magnets and there will be a pair of fixed coils connected in series when energized gives the same effect as that of the permanent magnets. In the field of these fixed coils there will be a moving coil which when energized acted upon by a torque by which it deflects

F_1 F_2: Fixed coils



M: Moving coil
R: High resistance in series with m
I_1 : load current
I_2: current through

The two fixed coils in series act as the current coil and the moving coil in series with R act as the potential coil. The moving coil is pivoted between the two fixed coils carries a current I_2 proportional to V. This current is fed to m through two springs which also provides the necessary controlling torque. This instrument can be used on both ac and dc circuits as both the coils are energized simultaneously by a common source due to which a unidirectional torque is produced.

RECOMMENDED QUESTIONS

1. Mention the different types of wiring. With relevant circuit diagrams and switching tables, explain the two – way and the three way control of lamps.
2. Explain the different types of wiring used in practice
3. Explain in brief the following:
   * Fuses
   * Specification of wires
   * Earthing and its necessity
4. Sketch any one type of earthing and indicate why such earthing of electrical equipments is necessary
5. With a neat sketch explain any one method of earthing electrical appliance.
6. With a neat circuit diagram, explain the two way control of a filament lamp.
7. Define domestic wiring. What important factors are to be considered in domestic wiring? Mention the difference types of wiring in practice.
8. What do you understand by "Earthing"? With a neat diagram explain plate earthing.
9. With a neat circuit diagram and a switching table, explain the two point control of a lamp.

12. With a neat sketch explain the pipe earthing method.

12. Explain the working principle of a fluorescent lamp when connected to an electrical supply source, with necessary auxiliary components and their functions

SOLUTION TO QUESTION PAPER

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1) With a neat sketch explain the pipe earthing method.

Sol: pipe earthing :

In this method of earthing a G.I pipe of 38 mm diagram and 2 meter (7 feet) length is embedded vertically into the ground. This pipe acts as an earth electrode. The depth depends on the condition of soil.

The earth wires are fastened to the top section of the pipe above the ground level with nut and bolts.

The pit area around the pipe is filled with salt and coal mixture for improving the condition of the soil and earthing efficiency. The schematic arrangement of pipe earthing system is shown in the Fig.10.

The contact surface of G.I. pipe with the soil is more as compared to the plate due to its circuit section and hence can handle heavier leakage current for the same electrode size.

According to Indian standard, the pipe should be placed at a depth of 4.75m. impregnating the coke with salt decreases the earth resistance. Generally alternate layers of salt and coke are used for best results.

In summer season, soil becomes dry, in such case salt water is poured through the funnel connected to the main G.I.pipe through19 mm diameter pipe. This keeps the soil wet.

The earth wires are connected to the G.I. pipe above the ground level and can be physically inspected from time to time. These connections can be checked for performing continuity tests. This is the important advantage of pipe earting over the plate earthing. The earth lead used must be G.I. wire of sufficient cross-sectional area to carry fault current safely. It should not be less than electrical equivalent of copper conductor of 12.97 mm^2 cross-sectional area.

The only disadvantage of pipe earthing is that the embedded pipe length has to be increased sufficiently in case the soil specific resistivity is of high order. This increases the excavation work and hence increased cost. In ordinary soil condition the range of the earth resistance should be 2 to 5 ohms.

In the places where rocky soil earth bed exists, horizontal strip earthing is used. This is suitable as soil excavation required for plate or pipe earthing is difficult in such places. For such soils earth resistance is between 5 to 8 ohms.

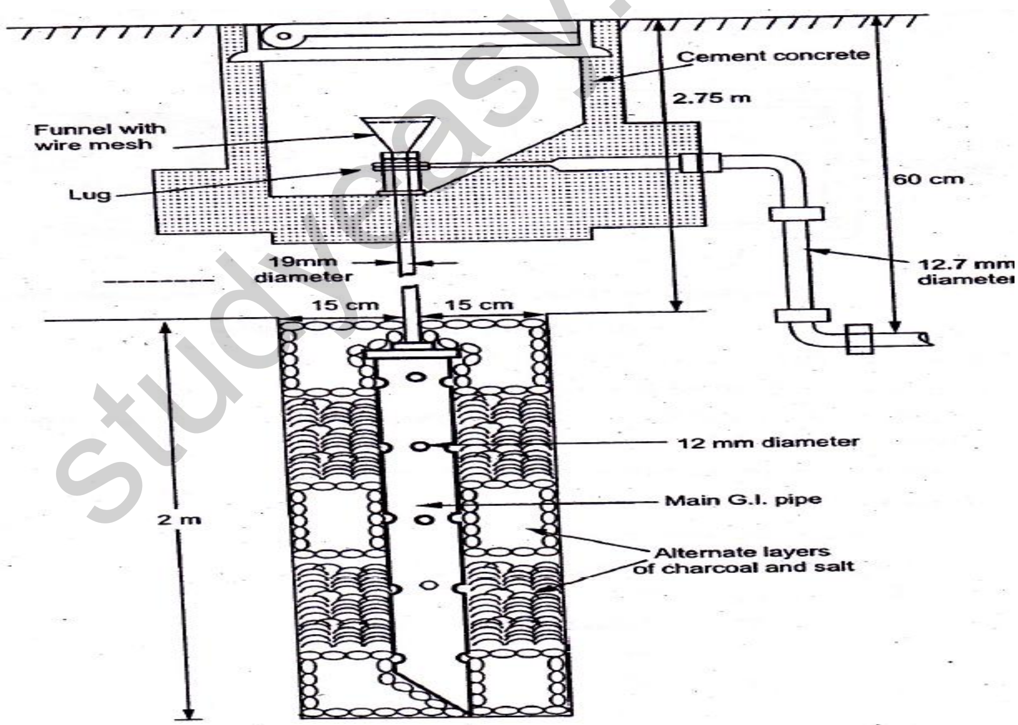


Fig. 10 Pipe earthing

2) with the help of a neat diagram. Explain the construction and principle of operation of single phase energy meter.

Sol. : Induction type single phase energy meter:

Induction type instruments are most commonly used as energy meters. Energy meter is an integrating instrument which measures quantity of electricity. Induction type of energy meters are universally used for domestic and industrial applications. These meters record the energy in kilo-watt-hours (k Wh).

Fig. 15 shows the inductance type single phase energy meter.

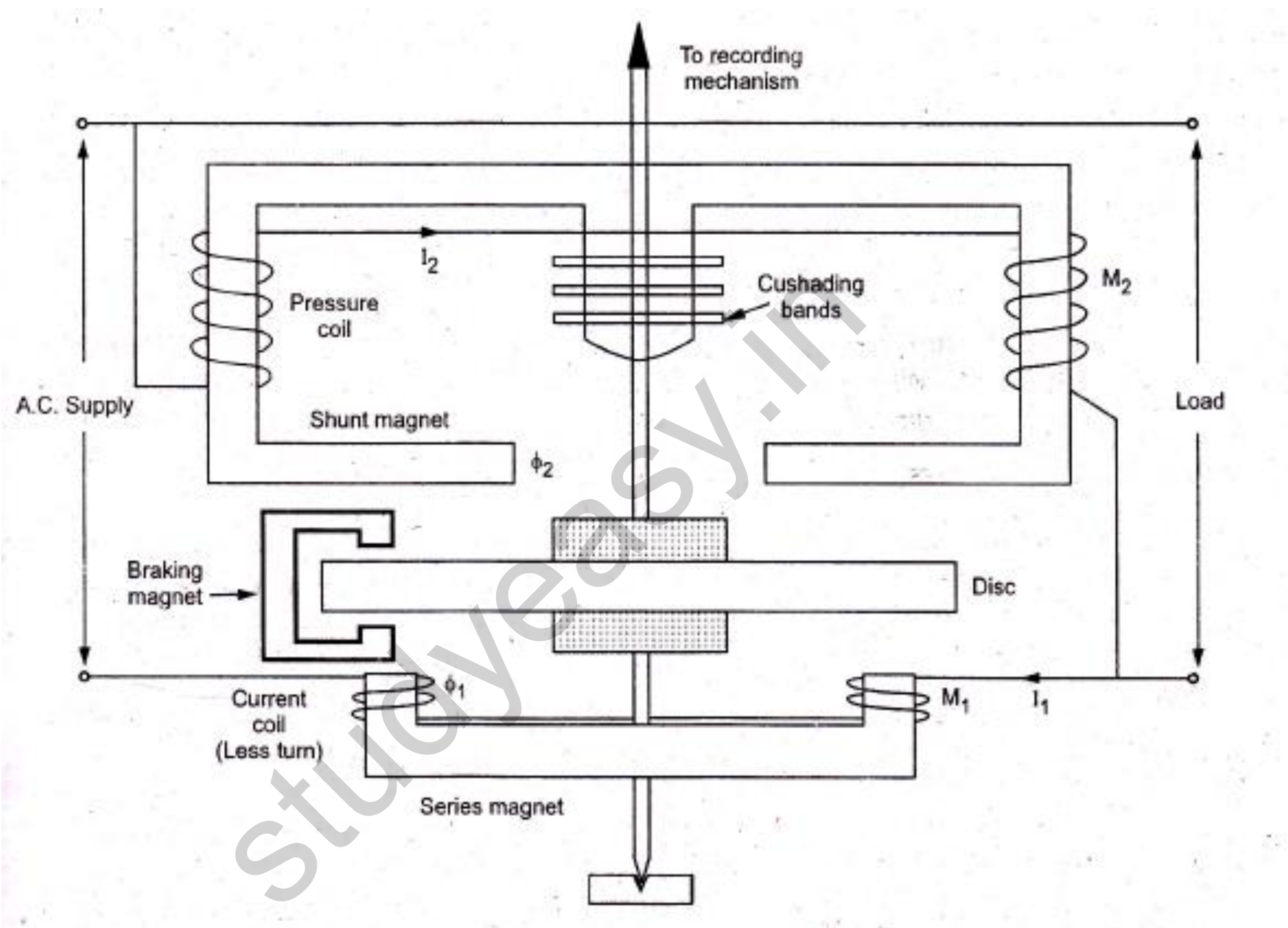


Fig. 15 Induction type single phase energy meter

It works on the principle of induction i.e. on the production of eddy currents in the moving system by the alternating fluxes. These eddy currents induced in the moving system interact with each other to produce a driving torque due to which disc rotates to record the energy.

In the energy meter there is no controlling torque and thus due to driving torque only, a continuous rotation of the disc is produced. To have constant speed of rotation braking magnet is provided.

Construction:

There are four main parts of operating mechanism

1) Driving system 2) moving room 3) braking system 4) registering system.

1) **Driving system:** It consists of two electromagnets whose core is made up of silicon steel laminations. The coil of one of the electromagnets, called current coil, is excited by load current which produces flux further. The coil of another electromagnetic is connected across the supply and it carries current proportional to supply voltage. This is called pressure coil. These two electromagnets are called as series and shunt magnets respectively.

The flux produced by shunt magnet is brought in exact quadrature with supply voltage with the help of copper shading bands whose position is adjustable.

2) **Moving system:** Light aluminium disc mounted in a light alloy shaft is the main part of moving system. This disc is positioned in between series and shunt magnets. It is supported between jewel bearings. The moving system runs on hardened steel pivot. A pinion engages the shaft with the counting mechanism. There area no springs and no controlling torque.
3) **Braking system:** a permanent magnet is placed near the aluminium disc for barking mechanism. This magnet reproduced its own field. The disc moves in the field of this magnet and a braking torque is obtained. The position of this magnet is adjustable and hence braking torque is adjusted by shifting this magnet to different radial positions. This magnet is called braking magnet.
4) **Registering mechanism:** It records continuously a number which is proportional to the revolutions made by the aluminium disc. By a suitable system, a train of reduction gears, the pinion on the shaft drives a series of pointers. These pointers rotate on round dials which an equally marked with equal division.

Working: since the pressure coil is carried by shunt magnet M_2 which is connected across the supply, it carries current proportional to the voltage. Series magnet M_1 carries current coil which carries the load current. Both these coils produced alternating fluxes Φ_1 and Φ_2 respectively. These fluxes are proportional to currents in their coils. Parts of each of these fluxes link the disc and induces e.m.f. in it. Due to these e.m.f.s eddy currents are induced in the disc. The eddy current induced by the electromagnet M_2 react with magnetic field produced by M_1 react with magnetic field produced by M_2. Thus each portion of the disc experiences a mechanical force and due to motor action, disc rotates. The speed of disc is controlled by the C shaped magnet called braking magnet. When disc rotates in the air gap, eddy currents are induced in disc which oppose the cause producing them i.e. relative motion of disc with respect to magnet. Hence braking torque T_b is generated. This is proportional to speed N of disc. By adjusting position of this magnet, desired speed of disc is obtained. Spindle is connected for ecording mechanism through gears which record the energy supplied.

3) Mention the different types of wiring. With relevant circuit diagrams and switching tables, explain two way and three way control of lamps.

Sol.: Types of wiring: Depending upon the above factors various types of wiring used in practice are:

5. Cleat wiring
6. Casing wiring
7. Surface wiring
8. Conduit wiring

i) Clear wiring:

In this type V.I.R or P.V.C wires are clamped between porcelain cleats.

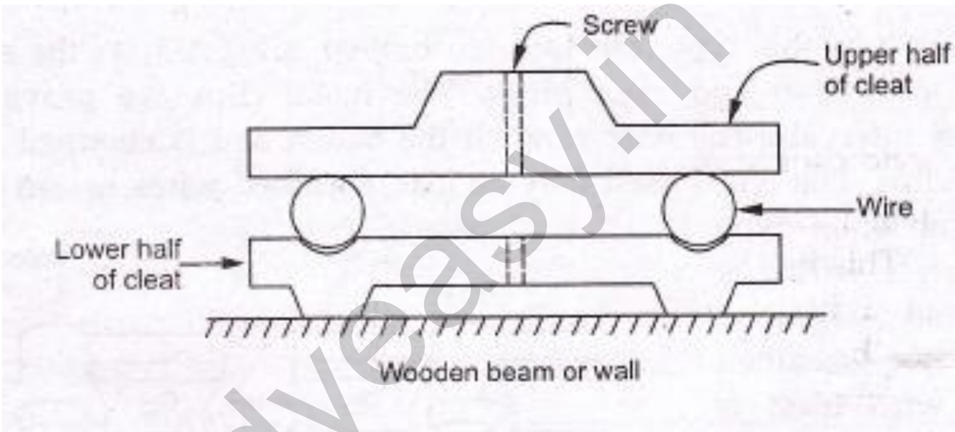


Fig. 16 Cleat wiring

The cleats are made up of two halves. One half is grooved through which wire passes while the other fits over the first. The whole assembly is then mounted on the wall or wooden beam with the help of screws.

This method is one of the cheapest method and most suitable for temporary work. It can be very quickly installed and can be recovered without any damage of material. Inspection and changes can be made very easily.

This method does not give attractive appearance. After some time due to sagging at some places, it looks shabby. Dust and dirt collects on the cleats. The wires are directly exposed to atmospheric conditions like moisture, chemical fumes etc. maintenance cost is very high.

Due to these disadvantages this type is not suitable for permanent jobs.

ii) Casing capping: This is very popularly used for residential buildings. In this method, casing is a rectangular strip made from teak wood or new a day's made up of P.V.C. It has two grooves into which the wires are laid. Then casing is covered with a rectangular strip of wood or P.V.C. of the same width, called capping. The capping is screwed into casing is fixed to the walls the help or porcelain discs or cleats.

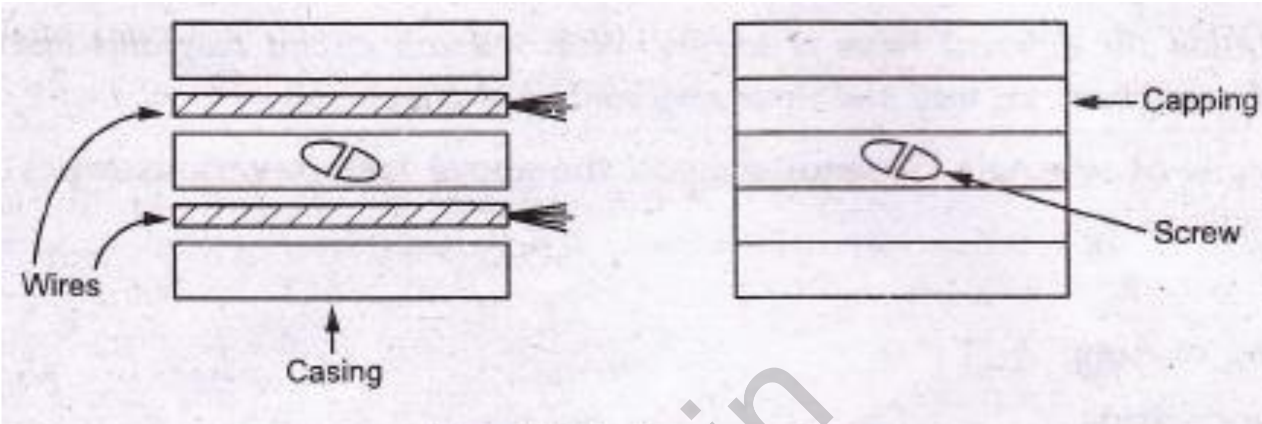


Fig. 17 Casing capping

Good protection to the conductors from dangerous atmospheric conditions, neat and clean appearance are the advantages of this type.

In case of wooden casing capping, there is high risk of fire along with the requirement of skilled labour. The method is costly.

Surface wiring: in this type, the wooden battens are fixed on the surface of the wall, by means of screws and rawl plugs. The metal clips are provided with the battens at regular intervals. The wire runs on the batten and is clamped on the batten using the metal clips. The wires used may lead sheathed wires or can tyre sheathed wires. Depending upon type of wire used surface wiring is also called lead Sheathed wiring or cab tyre sheathed wiring. If the wire used is though rubber Sheathed then it is called T.R.S. wiring While if the wire used is cab tyre Sheathed Then it is called C.T.S wiring.

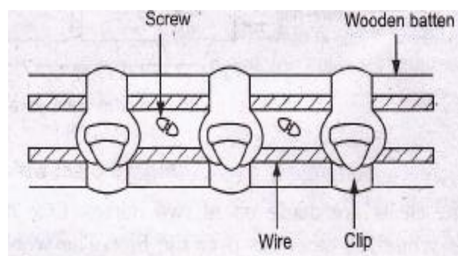


Fig. 18 Wooden batten wiring

Conduit wiring: In this method, metallic tubes called as conduits are used to run the wires. This is the best system of wiring as it gives full mechanical protection to the wires. This is most desirable for workshops and public Buildings. Depending on whether the conduits are laid inside the walls or supported on the walls, there are two types of conduit wiring

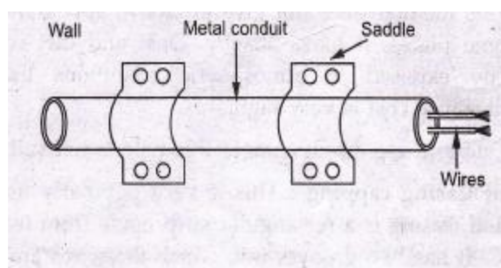


Fig. 19 Surface conduit wiring

which are :

iii) Surface conduit wiring: in this method conduits are mounted or supported on the walls with the help of pipe books or saddles. In damp situations, the conduits are spaced apart from the wall by means of wooden blocks.

iv) Concealed conduit wiring: In this method, the conduit are buried under the wall aty the some of plastering. This is also called recessed conduit wiring.

The beauty of the premises is maintained due to conduit wiring. It is durable and has long life. It protects the wires from mechanical shocks and fire hazards. Proper earthing of conduits makes the method electrical shock proof. It requires very less maintenance.

The repairs are very difficult in case of concealed conduit wiring. This method is most costly and erection requires highly skilled labour. These are few disadvantages of the conduit type of wiring. In concealed conduit wiring, keeping conduit at earth potential is must.

5) Explain two way & three way control of lamps

Two way control of lamps:

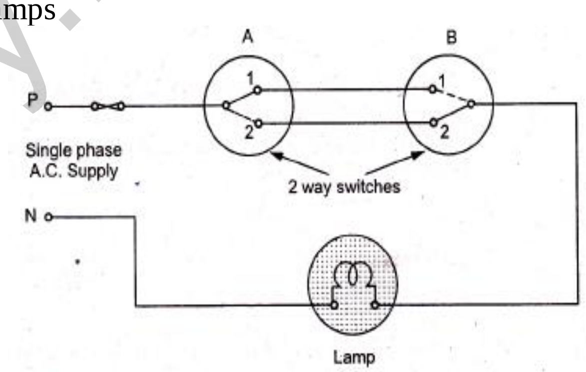


Fig. 20 Control of one from two points

Thisis also called as staircase wiring as it is commonly used for stair cases and corridor lighting. It consist of two way switches. A two way switch operatesalways in one of the two possible positions.The circuits is shown in the fig. 20.

Assume that lamp is on first floor. Switch A In on first floor and B in on second floor. In the position shown, the lamp is OFF.

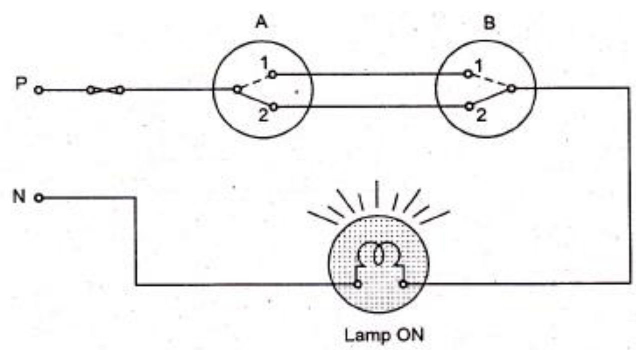


Fig. 21 ON position of lamp

When person changes position of Switch A from (1) to (2) then lamp gets phase through Switches A and B it gets switched ON as shown In fig. 21.

When person reaches on second floor, the lamp is required to be switched OFF. So person will change switch B from (2) to (1), due to which phase connection reaching to the lamp gets opened and lamp will switched OFF as shown in fig. 22.

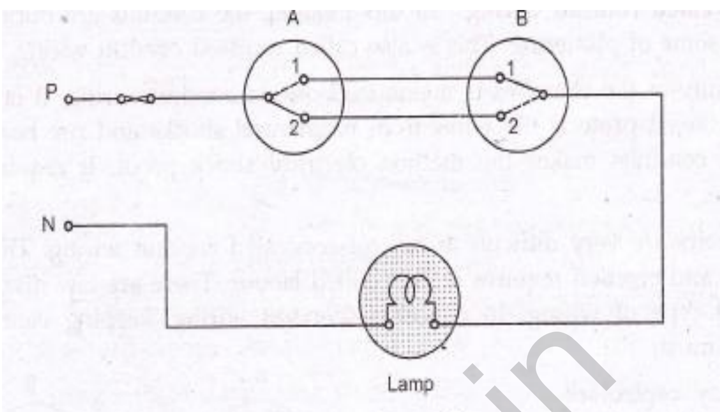


Fig. 22 OFF position of lamp

Switch A position	Switch B position	`Switch C position
1	1	ON
2	2	ON
2	1	OFF
1	2	OFF

3) Explain **Three way control of lamps:** This is also a type of stair case wiring. It consist of two way switches A and B and one intermediate switch C. The circuit used to have three way control of lamps is shown in the fig. 23.

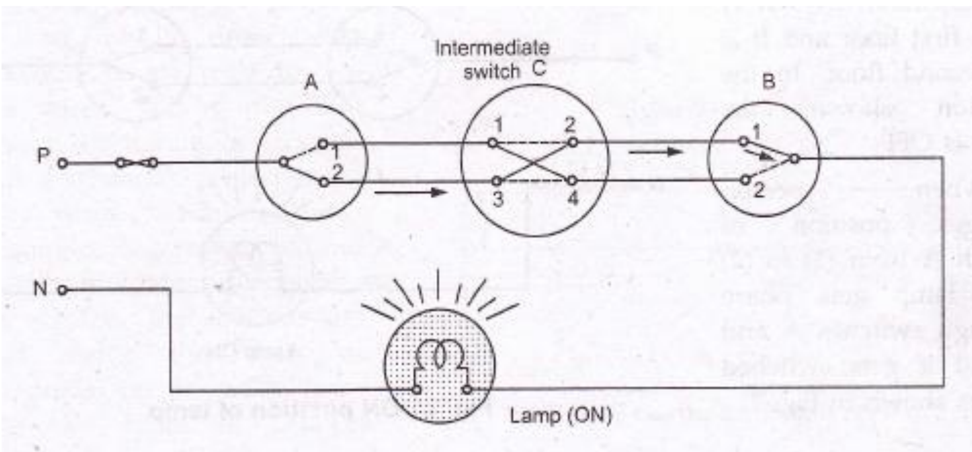


Fig. 23

The intermediate switch can have positions to connect point 1 – 4, 3 – 2, as shown or 1 – 2 and 3 – 4 shown dotted. The switch A is on the first floor and switch B is on third floor say.

In the position shown is, fig. 23, the lamp is ON.

When person from floor 2 changes switch C position to have connections 1 – 2 and 3 – 4 then it can be seen from fig. 24 that there is open circuit in the connection and lamp gets switched OFF.

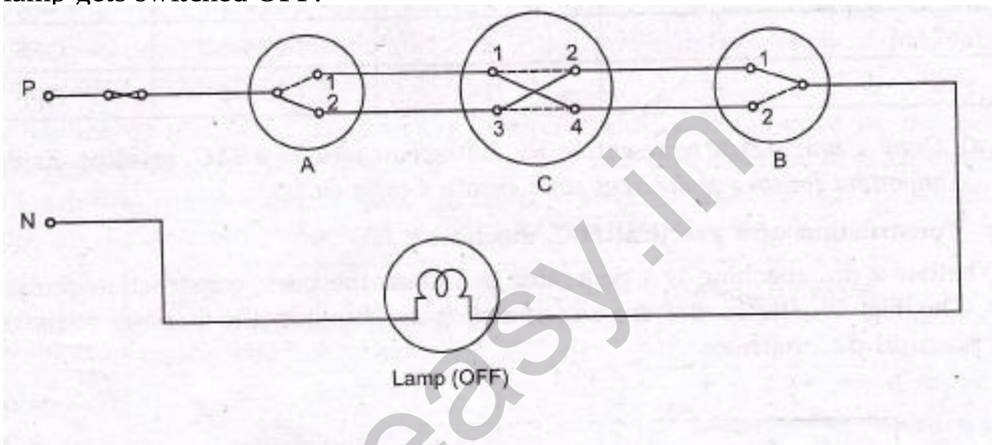


Fig. 24

Now if the person from the third floor changes the position of switch B from 1 to 2, then again lamp gets supply through position 2 of A, 3 – 4 of C and 2 of B shown in the fig. 25. The lamp gets switched ON.

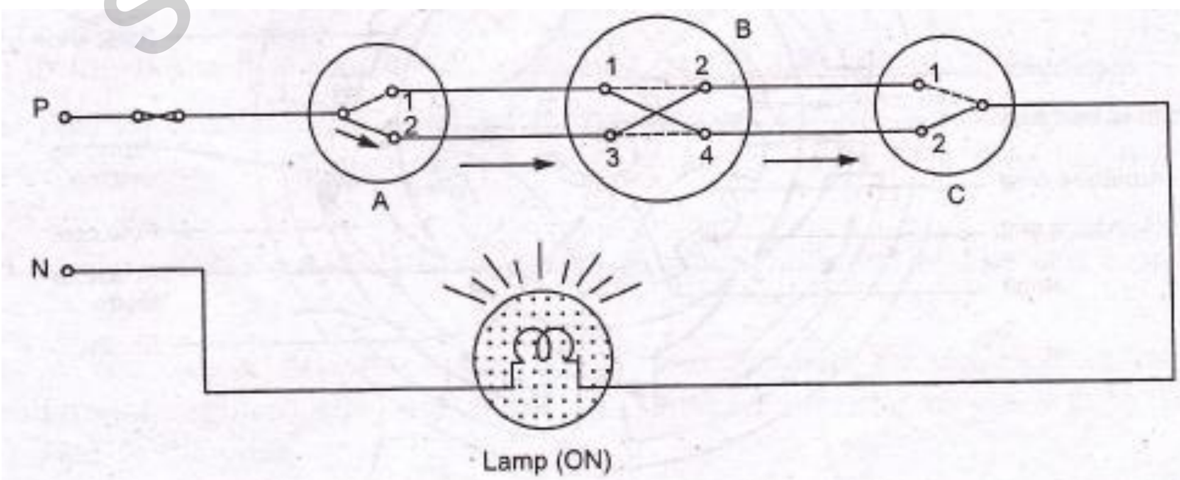


Fig. 25

Sr. no.	**Switch A position**	**Switch B position**	**Switch C position**	**Condition of lamp**
1	**1**	**1**	**1 – 4, 3 – 2**	**OFF**
2	**2**	**2**	**1 – 4, 3 – 2**	**OFF**
3	**1**	**2**	**1 – 4, 3 – 2**	**ON**
4	**2**	**1**	**1 – 4, 3 – 2**	**ON**
5	**1**	**1**	**1 – 2, 3 – 4**	**ON**
6	**2**	**2**	**1 – 2, 3 - 4**	**ON**
7	**1**	**2**	**1 – 2, 3 – 4**	**OFF**
8	**2**	**1**	**1 – 2, 3 – 4**	**OFF**

Unit-5:

DC Machines: Working principle of DC machine as a generator and a motor. Types and constructional features. emf equation of generator, relation between emf induced and terminal voltage enumerating the brush drop and drop due to armature reaction. Illustrative examples.
DC motor working principle, Back emf and its significance, torque equation. Types of D.C. motors, characteristics and applications. Necessity of a starter for DC motor. Illustrative examples on back emf and torque.

UNIT-5

D.C. MACHINES.

Working principle of D.C.Machine as generator and motor, constructional features, EMF equation of generator and simple problems, back emf and torque equation of DC motors, simple problems, types of DC motors, characteristics and applications, necessity of starter, 3-point starter.

A machine which works on direct current is defined as a **D.C.Machine**.

D.C.Machines are of two types. **(i) D.C.Generator and (ii) D.C.Motor.**

Sl. No.	**D.C. Generator**	**D.C.Motor**
1	**Definition:** A generator is a rotating machine which converts mechanical energy into electrical energy	**Definition:** A motor is a machine which converts electrical energy into mechanical energy
2	**Principle:** Whenever a coil is rotated in a magnetic field an e.m.f. will be induced in this coil and is given by $e=BlvSin\theta$ volts/coil side where, B=The flux density in Tesla, l=the active length of the coil side in meters, v=the velocity with which the coil is moved in meters/sec and θ is the angle between the direction of the flux and the direction of rotation of the coil side.	**Principle:** Whenever a current coil is placed under a magnetic field the coil experiences a mechanical force, and is given by $F= BIlSin\theta$ Newtons/coil side. Where, I is the current through the coil in ampere.
3	The direction of the emf induced is fixed by applying the Fleming's right hand rule	The direction of the force acting is fixed by applying the Fleming's left hand rule.

CONSTRUCTION OF A D.C.MACHINE.

Salient parts of a D.C.machine are:

- Field system (poles)
- Coil arrangement (armature)
- Commutator
- Brushes
- Yoke

Fig shows the details of a four pole D.C. machine with both shunt and series field windings.
Field system: This is made of electromagnets, wherein a iron laminated core is wound with well insulated enameled copper wire. The core is laminated to minimize the eddy current loss. Each lamination is dipped in varnish and dried. A pole shoe is attached to the pole face to direct the flux to concentrate radially on to the armature thereby reducing the leakage and fringing flux. Poles are fixed to the yoke by means of bolts.
Armature: This is the rotating part of the machine made of laminated iron core cylindrical in structure with slots on its periphery. Insulated copper coils are laid in these slots, and these coils are connected for lap or wave connection. The core laminations are firmly mounted on a shaft fitted with smooth bearings on either side for smooth rotation.

Comparison of lap and wave windings:

LAP	WAVE
Number of armature parallel paths is equal to the number of poles.	Number of parallel paths is always equal to two.
Preferred when large current at lesser voltage is the requirement.	Preferred when large voltage with lesser current is the requirement.

Commutator: As the induced e.m.f. in the armature is alternating commutator converts alternating e.m.f. into unidirectional e.m.f.
This is cylindrical in structure made of copper segments with mica insulation between them and is firmly fixed on to the shaft carrying the armature and the armature coil free ends are brazed to the commutator segments.
Brushes: These are current collecting devices placed on the body of the commutator with a holder. Brushes are made of carbon, copper or graphite.
Yoke: This is the outer most part of the machine made of cast steel which is the mechanical enclosure for the machine to protect it from dust and moisture and also provides the return path for the magnetic flux and carries half the flux per pole.

E.M.F. Equation:
Let the D.C. machine has **P** number of poles, **Z** number of armature conductors arranged in **A** number of parallel paths. Let **Φ** be the flux per pole and **N** is the speed of rotation in revolutions per minute.
Consider one North Pole of the machine under which a group of armature conductors all being connected in series. Let **x** be the spacing between any two neighboring conductors ant **t** be the time taken to move through this distance of **x**.

The total flux per pole **Φ** is made of several lines and one line of flux is cut by one conductor when it moves through a distance of **x** in **t** seconds.

Therefore the induced emf in the 1st conductor when cut by the flux of Φ_1 is

$$e_1 = \Phi_1/t \quad \text{volts}$$

Similarly in the 2nd conductor $e_2 = \Phi 2/t$ volts, and so on.

Therefore the total emf induced in all the conductors under one pole is the sum of all these emf's.

$$E = e_1 + e_2 + e_3 + e_4 + \ldots\ldots$$

$$E = \Phi_1/t + \Phi 2/t + \Phi 3/t + \Phi 4/t + \ldots\ldots$$

$$E = \Phi/t \quad \text{volts/pole.}$$

For all the P number of poles $E = P\Phi/t$ volts

The speed is defined as **N** revolutions per minute,

N revolutions in **one minute** or **60 seconds**.

1 revolutions will be in time of **60/N** seconds, and as one revolution corresponds to all the **Z** number of conductors the time **t** for a travel of distance **x** can be written as **t= 60/NZ** seconds.

Therefore the induced EMF $E = P\Phi/t = P\Phi/60/(NZ) = PZN\Phi/60$.

As the Z number of conductors are arranged in A number of parallel paths,

The induced e.m.f per parallel path is

$$E = PZN\Phi/60A \text{ volts.}$$

As **P, Z, A** are fixed the induced e.m.f is mainly dependent on the flux and the speed, and hence we write that the induced e.m.f **E** is proportional to the product of the speed **N** and the flux **Φ.**

Types of D.C. Generators.

D.C. Generators are classified on the basis of the method of exciting the field coils as **(i) Separately excited generators and**

(ii) Self excited generators.

In separately excited type the field coils are excited from an independent D.C. source.

In self excited type excitation of the field coils are done by feeding back a part of the output of the generator.

Self excitation can be done in three ways:

- **(i)** By connecting the field coils **across** the armature- **Shunt excitation** (Fig 1.)
- **(ii)** By connecting the field coils in **series** with the armature- **Series**. (Fig 2) **excitation.**
- **(iii)** By using both the shunt and series field coils together- **Compound excitation.**

In compound excitation the fluxes due to the shunt field and the series field may **support** each other or **oppose** each other and accordingly they are called **Cumulatively** compounded or **Differentially** compounded generators. (Figs. 3 and 4.)

There are **two** more ways of connecting the shunt and series field coils with the armature: **(i) Long shunt connection**:- Here the armature and the series field coils are connected in series and the shunt field circuit is connected in parallel with this combination. (Fig. 5.)
(ii) Short shunt connection: - Here the armature and the shunt field circuits are connected in parallel and the series field coils are connected in series with this combination.(Fig. 6)

A 500V shunt motor has 4 poles and a wave connected winding with 492 conductors. The flux per pole is 0.05 Wb. The full load current is 20 Amps. The armature and shunt field resistances are 0.1 Ω and 250Ω respectively. Calculate the speed and the developed torque.

Sol:

P = 4, V =500volts

$\Phi = 0.05$Wb

Z = 492

$I_L = I_{F.L.} = 20$A

$R_a = 0.1\Omega$

$R_{sh} = 250\Omega$, A = 2 for wave winding

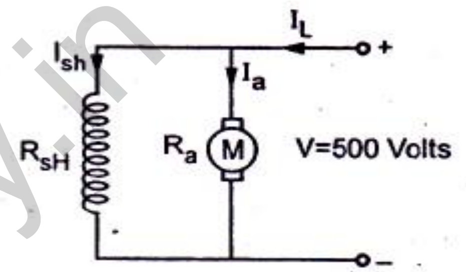


Fig. 11

$$I_{sh} = \frac{V}{R_{sh}} = \frac{500}{250} = 2A$$

$I_L = I_a + I_{sh}$ $\therefore I_a = I_L - I_{sh} =$

Now, $E_b = \frac{\phi ZNP}{60A} = \frac{0.05 \times 492 \times N \times 4}{60 \times 2}$

But, $E_b = V - I_a R_a = 500-(18)(90.1) = 500-1.8$

$$\therefore \quad 498.2 = \frac{0.05 \times 492 \times N \times 4}{60 \times 2}$$

$$\therefore \quad N = \frac{498.2 \times 2 \times 60}{492 \times 0.05 \times 4}$$

$\therefore$ N = 607.56 rpm

Now, $E_b \cdot I_a = Ta \times \omega$

$$T_a = \frac{E_b \cdot I_a}{\omega} = \frac{498.2 \times 18}{\left[\frac{2\pi \times 607.56}{60}\right]}$$

$$T_a = \frac{498.2 \times 18 \times 60}{2\pi \times 607.56}$$

$T_a = 140.94$N-m) A 250 KVA, 11000/415V, 50 Hz single phase

Questions and Problems on D.C. Generators:

1. Draw the cross sectional views of a typical 4 pole D.C. machine and explain the function of each part.
2. With usual notations derive expression for the induced emf of a D.C. machine.
3. A 4 pole generator with wave wound armature has 51 slots, each having 24 conductors. The flux per pole is 0.01 wb. At what speed must the armature be rotated to

give an induced emf of 220 volt. What will be the voltage developed if the winding is lap and the armature rotates at the same speed.

4. Find the useful flux per pole of a 250 V, 6pole shunt generator having two circuit connected armature winding with 220 conductors. At normal temperature the overall armature resistance including brushes is 0.2 ohm. The armature current is 13 A and the speed is 910 rpm.

5. A 4 pole, 100V, shunt generator with lap connected armature winding having a field and armature resistances of 50 and 0.1 ohm respectively supplies sixty, 100V, 40 watt lamps. Calculate the total armature current, current per parallel path and the generated emf.

6. A long shunt cumulatively compounded D.C. generator supplies 7.5 KW power at 250V. The shunt field, series field and armature resistances are respectively 125 ohm, 0.3 ohm and 0,5 ohm. Calculate the induced emf.

7. In a 110V compound generator the resistances of the armature, shunt and series field coils are 0.06, 25 and 0.04 ohms respectively. The load consists of 200 lamps each rated at 55 Watt, 110 Volt. Find the generated emf when connected for (i) Long shunt and (ii) Short shunt.

D.C. MOTOR.

Principle: Whenever a current coil is placed in a magnetic field the coil experiences a mechanical force, and is given by

$F = BIl\sin\theta$ newtons. Where B is the flux density in Tesla
I is the current through the coil
l is the active length of the coil side
θ is the angle between the movement of the coil and the direction of the flux

The direction of the force acting can be decided by applying Fleming's left hand rule.

The construction of a D.C.Motor is same as the construction of a D.C.generator.

Types of D.C.Motors:

Depending on the interconnection between the armature and the field circuit D.C.Motors are classified as (i) Shunt Motor, (ii) Series Motor and (iii) Compound motors just like D.C.Generators.

Back EMF: Whenever a current coil is placed under a magnetic field the coil experiences a mechanical force due to which the coil starts rotating. This rotating coil again cuts the magnetic lines of force resulting an EMF induced in it whose direction is to oppose the applied EMF (as per Fleming's right hand rule), and hence the name **BACK EMF** or **Counter Emf.**

Significance of Back EMF: Back EMF is a must in a motor which helps to regulate the armature current and also the real cause for the production of torque.

Expression for the back Emf is given by **$E=V-IaRa$,**
Where E is the back emf, V is the applied emf, Ia is the armature current and Ra is the armature circuit resistance. And also **$E= PZN\Phi/60A$** volts, from the machine parameters.

Production of torque in a D.C. Motor.

The production of torque in a d.c. motor can be well explained with the help of the following figures.

Fig (a) represents the magnetic field distribution between a bipolar magnet from North pole to South pole.
Fig(b) shows the field set up around a current carrying coil
In fig © the current carrying coil is brought under the influence of bipolar magnetic field.
The resultant field around the coil due to the inter action of the main field and the coil field is seen in fig (d) where in the flux is strengthened in the left part of the upper coil side and weakened in the right part of the upper coil side and vice-versa in the lower coil side. The resultant flux which strengthened at one point exerts a force on the conductor as per Fleming's left hand rule and thereby the coil side experiences a mechanical force.

In the construction it is seen that several coils sides are on the armature and the tangential force acting on each of these coil sides add each other and resulting in a unidirectional movement which makes the armature to rotate at a uniform speed thereby torque is produced.

TORQUE EQUATION:

Let P be the total number of poles, Z be the total number of armature conductors arranged in A number of parallel paths. Let **Φ** be the flux per pole, N be the speed of rotation in rpm, and T be the torque in Nm.

We know that the back emf **$E=V-IaRa$**
Multiplying the above equation by Ia on both sides
We get **$EIa=VIa-Ia^2 Ra$**
Where VIa represents the Power input to the armature, Ia2 Ra represents the armature copper loss and EIa represents the Total power output of the armature which is the electrical power converted into mechanical power called the electro-mechanical power
in watts. The equivalent mechanical power is given by $2 \pi NT/60$ watts.

Therefore, **$EIa=2 \pi NT/60$ watts**
But $E = PZN \Phi/60A$, therefore the torque **$T= PZ\Phi Ia/2\pi A$** Nm**.**
From the above equation it can be seen that the torque is directly proportional to the product of the flux and the armature current.

Speed of a D.C.Motor:-

We know that for a motor in general the back emf e is given by

E= V-IaRa=PZNΦ/60A

From which we write,

N=(V-IaRa)/PZΦ60A,

and the speed N is proportional to **(V-IaRa)/Φ,**

From the above equation we write the speed is directly proportional to the applied voltage V, and the armature current Ia and inversely proportional to the flux Φ.

Characteristics of D.C.Motors: To study the performance of a motor it is necessary to study the variation of its speed and torque with the variations of the load on it.

There are two types of characteristics: (i) Speed v/s load characteristics
(ii) Torque v/s load characteristics

(i) **Speed/Load characteristics: (a) D.C.Shunt Motor**:

In a shunt motor the flux is considered to be constant because of the reason that the field circuit is connected across a constant power supply. Also as the applied voltage is constant the speed is directly proportional to the armature current only, and also as the load is increased the armature current also increases at the same rate and the speed becomes constant. But due to the increased friction at the bearings with the increase of the load there is a small decrease in the speed. The characteristic is shown in the fig. and is compared with the ideal characteristics. The drop in the speed can be reduced by slightly de-exciting the field flux, there by the speed is controlled.

(b) Series Motor:

In a series motor the flux is solely dependent on the armature current hence the speed variation with load is not like shunt motor. At no load condition only residual flux is in action which is very very small resulting in a dangerously high speed. Therefore series motors are not to be started on no load, which result in the initial speed of dangerously high value called **RUN AWAY SPEED** which severely damages the motor. Hence in series motors there is a provision of a fly wheel fixed to the shaft which acts like a mechanical load to prevent the motor to attain this high speed.

characteristics of D.C. i) series and ii) shunt motors. Mention two applications of each motor.

DC series motor :

i) N=1 characteristics :

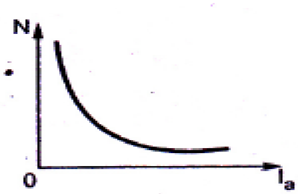


Fig. 2 N Vs I_a for series motor

ii) T-I characteristics :

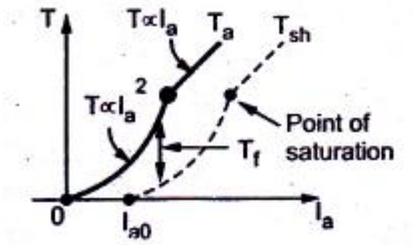


Fig. 3 T Vs I_a for series motor

Applications : Cranes, trolleys .

DC shunt motor :

i) N-I characteristics :

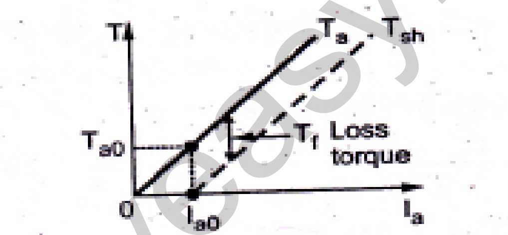


Fig. 4 T Vs I_a for shunt motor

ii) T-I characteristics :

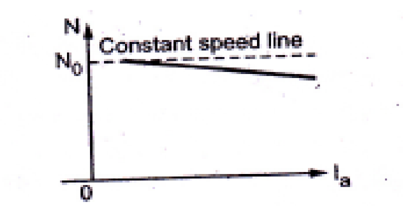


Fig. 5 N Vs I_a for shunt motor

Applications : Lathe machine, milling machine.

b) Give reasons for the following in an alternator –

i) Armature is stationary field is rotating .

ii) Distributed winding is used instead of concentrated winding. (6)

Sol.: i) Armature is stationary field is rotating –

a) The stationary armature can be easily insulated for high a.c. generators voltages.

b) The armature winding can be braced easily to prevent deformation.

c) The armature winding can be directly connected to load. Thus load current does not pass through brush contain.

d) The excitation voltage is low.

ii) Distributed winding is used instead of concentrated winding –

In distributed type of winding all the coils belonging to a phase are well distributed over the 'm' slots per phase, under every pole. Distributed makes the waveform of the induced e.m.f. more sinusoidal in nature. Also in concentrated winding due to large number of conductors per slot, heat dissipation is poor . So in **practice, double layer, short pitched and distributed type of armature winding is preferred for the alternators.**

A 120V DC shunt motor has an armature resistance of 60Ω. It runs at 1800 RPM, when it takes full load current of 40A. Find the speed of the motor while it is operating at half the full load, with load terminal voltage remaining same.

Sol.:

$$I_f = \frac{120}{60} = 2A \quad , \qquad I_{L1} = 40\ A$$

$I_{L1} = I_{a1} + I_f$

$Ia_1 = I_{L1} - I_f = 40 - 2 = 38A$

$$E_{b1} = V - I_{a1}R_a = 120 - (38)(0.2)$$

$= 120 - 7.6 = 112.4V$

$N_1 = 1800rpm$

Now for shunt machine

$T \propto I_a$ as $\emptyset$ remains constant.

For half load

$$T_2 = \frac{1}{2} T_1$$

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$$

$$\frac{\frac{1}{2}T_1}{T_1} = \frac{I_{a2}}{I_{a1}}$$

$I_{a2} = ½ I_{a1} = ½ (38) = 19A$

$E_{b2} = V - I_{a2}\ Ra = 120 - (19)(0.2) = 120 - 3.8$

$E_{b2} = 116.2V$

Now, $E_b \propto N$ for

Φ constant

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$

$$\frac{116.2}{112.4} = \frac{N_2}{1800}$$

$$N_2 = 116.\frac{2}{112}.4 \times 1800$$

$N_2 = 1860.85$ rpm

A 250 V d.c. shunt motor has an armature resistance of 0.5 Ω and shunt field resistance of 250 Ω.When driving a load at 600 rpm., the torque of which is constant, the armature takes 20 A. If it is desired to raise the speed from 600 to 800 r.p.m., what resistance must be inserted in the field circuit? Assume the magnetization curve to be a straight line. [10]

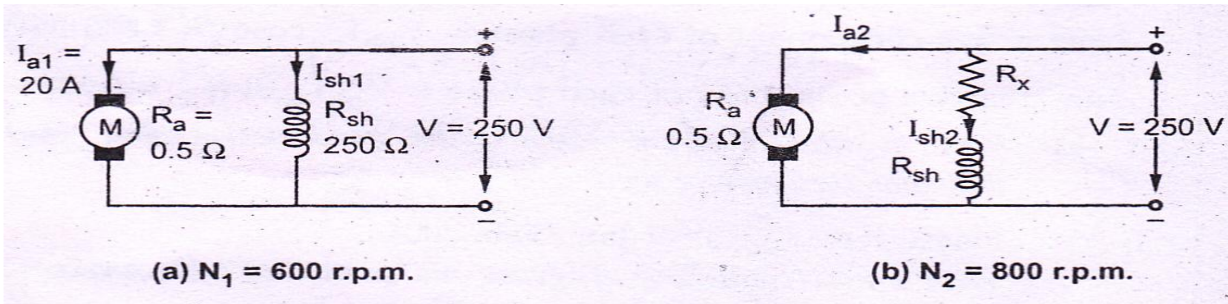


Fig. 6

Ans. :

$I_{sh1} = \frac{V}{R_{sh}} = \frac{250}{250} = 1A$

$E_{b1} = V - I_{a1}R_a = 250 - 20 \times 0.5 = 240$ V

$T \propto \phi I_a \propto I_{sh} I_a$

$\therefore \frac{T_1}{T_2} = \frac{I_{sh1}}{I_{sh2}} \times \frac{I_{a1}}{I_{a2}} = 1$ … Torque is constant

$\therefore \frac{1}{I_{sh2}} \times \frac{20}{I_{a2}} = 1$ i.e. $I_{a2} I_{sh2} = 20$

$N \propto \frac{E_b}{\phi} \propto \frac{E_b}{I_{sh}}$

$\therefore \frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{sh2}}{I_{sh1}}$

$\therefore \frac{600}{800} = \frac{240}{E_{b2}} \times \frac{I_{sh2}}{1}$

$\therefore \frac{E_{b2}}{I_{sh2}} = 320$ … (2)

But $E_{b2} = V - I_{a2} R_a = 250 - 0.5 I_{a2}$

$\therefore E_{b2} = 250 - 0.5\left[\frac{20}{I_{sh2}}\right]$ … From (1)

Using in (2) , $\frac{250 - \frac{10}{I_{sh2}}}{I_{sh2}} = 320$

$\therefore$ $250 I_{sh2} - 10 = 320 I_{sh2}^2$ i.e. $320 I_{sh2}^2 - 250 I_{sh2} + 10 = 0$

Solving, I_{sh2} = 0.7389 A, 0.0422 A

Neglecting lower value, I_{sh2} = 0.7389 A

But, $I_{sh2} = \frac{V}{R_{sh} + R_X}$ i.e. $0.7389 = \frac{250}{250 + R_X}$

$\therefore$ $\mathbf{R_X = 88.3407\ \Omega}$ … External resistance required

Recommended Questions and problems on D.C.Motors:

1. Fundamentally, upon what two quantities does the speed of the motor depend? Derive the equation which gives the speed in terms of terminal voltage, armature resistance drop, and the flux per pole.
2. When load is applied to a motor what is its first reaction? With the shunt Motor, how does this reaction affect the counter emf? The current to the armature?
3. How does the flux in series motor vary with the load current? Show the relation of internal torque to load current, assuming no saturation in the magnetic circuit. Show the relation of speed to load current. What precautions should be taken when the series motor is being installed for industrial drives?
4. In what way do the windings of a compound motor differ from those of a shunt motor and a series motor? In what two ways, with respect to the shunt winding, may the series winding be connected?
5. Discuss the speed characteristic and torque characteristic of the cumulative-compound motor. What is the advantage of this motor over the series motor?
6. Why is a starting resistance necessary for D.C. motors? With the shunt motor, in what circuit the starting resistor is connected? Why should it not be connected in the line?
7. The resistance of the armature of a 25-hp 240-volt shunt motor is 0.083 ohm. When connected to a 240-volt supply the armature develops a counter emf of 232.8-volts. Determine: (a) armature current; (b) armature current when connected across same power supply while stationary; (c) counter emf when armature current is 110 amperes.
8. A 230-volt 4-pole 15 hp shunt motor has 702 conductors connected for simplex wave and its resistance is 0.252 ohm. The flux per pole is 7.65 milliwebers and the armature current is 60-Amp. Determine speed of armature.
9. A four pole d.c. shunt motor has 456 surface conductors connected in simplex wave. The flux is 2.41 milliwebers per pole (a) Determine the counter emf when the speed is 1500 rpm. The armature resistance is 0.2 ohm. (b) Determine the terminal voltage when armature current is 60-amp if the speed and flux remain constant.
10. A 60-hp, 250-volt, 1200 rpm shunt motor takes 214 amp at 250 volts. The field current is 1.05 amp, and the combined total armature resistance is 0.039 ohm. The motor speed when running light is 1200 rpm, and the line current is 8.6 amp. Determine the internal power and torque developed.
11. A 25-hp, 250-volt d.c. series motor has its armature and series field resistance of 0.12 ohm and 0.10 ohm respectively. When the motor takes 85 amp, the speed is 600 rpm. Determine the speed when the current is (a) 100 amp, (b) 40 amp. Assume saturation curve is a straight line, and neglect armature reaction.
12. A 6 pole d.c. generator runs at 850 rpm, and each pole has a flux of 0.2 milliwebers. If there are 150 conductors in series between each pair of brushes, what is the value of the generated emf?
13. A 220-volt shunt motor has a field resistance of 400 ohm and an armature resistance of 0.1 ohm. The armature current is 50 amps, and the speed is 900 rpm. Assuming a straight line magnetization curve, calculate (a) the additional resistance in the field to increase the speed to 1000 rpm for the same armature current, and (b) the speed with the original field current and an armature current of 200 amps.

SOLUTION TO QUESTION BANK

1) A 500V shunt motor has 4 poles and a wave connected winding with 492 conductors. The flux per pole is 0.05 Wb. The full load current is 20 Amps. The armature and shunt field resistances are 0.1Ω and 250Ω respectively. Calculate the speed and the developed torque.

Sol:

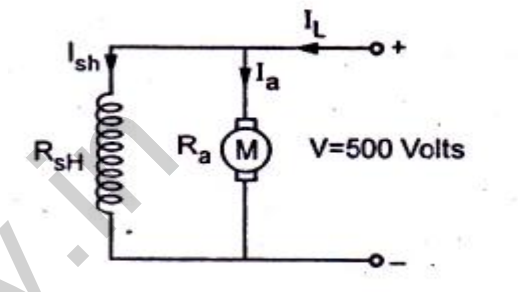


Fig. 11

$P = 4$, $V = 500$volts

$\Phi = 0.05$Wb

$Z = 492$

$I_L = I_{F.L.} = 20$A

$R_a = 0.1\Omega$

$R_{sh} = 250\Omega$, $A = 2$ for wave winding

$$I_{sh} = \frac{V}{R_{sh}} = \frac{500}{250} = 2A$$

$$I_L = I_a + I_{sh} \qquad \therefore I_a = I_L - I_{sh} =$$

Now, $$E_b = \frac{\phi ZNP}{60A} = \frac{0.05 \times 492 \times N \times 4}{60 \times 2}$$

But, $$E_b = V - I_a R_a = 500-(18)(90.1) = 500-1.8$$

$$\therefore \quad 498.2 = \frac{0.05 \times 492 \times N \times 4}{60 \times 2}$$

$$\therefore \quad N = \frac{498.2 \times 2 \times 60}{492 \times 0.05 \times 4}$$

$$\therefore \quad N = 607.56 \text{ rpm}$$

Now, $$E_b \cdot I_a = T_a \times \omega$$

$$T_a = \frac{E_b \cdot I_a}{\omega} = \frac{498.2 \times 18}{\left[\frac{2\pi \times 607.56}{60}\right]}$$

$$T_a = \frac{498.2 \times 18 \times 60}{2\pi \times 607.56}$$

$$T_a = 140.94 \text{N-m}$$

$$T_g = 0.159 \phi I_a \cdot \frac{PZ}{A}$$

$$= 0.159 \times 0.05 \times 18 \times \frac{4 \times 492}{2}$$

$$\therefore \quad T_g = 140.81 \text{N-m}$$

2)A 4 pole , 1500 r.p.m d.c generator has a lap wound armature having 24 slots With 10 conductors per slot. If the flux pole is 0.04 Wb, calculate the e.m.f generated in the b armature. What would be the generated e.m.f if the winding is wave connected ? **(6)**

Ans: $P = 4$, $N = 1500$ r.p.m, Lap i.e. $A = P$, $\Phi = 0.04$ Wb

Z = Slots x Conductors / Slot = 24x10= 240

$\therefore \quad E_g = (PNZ/60A = \dfrac{0.04x4x150_0x24_0}{60 \times 4} = \mathbf{240\ V}$

If winding is wave connected, A=2

$\therefore \quad E_g = \dfrac{0.04x4x150_0x24_0}{60 \times 4} = \mathbf{480\ v}$

3) Derive an expression for torque a d.c motor **(4)**

d) The current drawn from the mains by a 220 V D.C shunt motor is 4 A on no- load The resistance and armature windings are 110 ohm and 0.2 ohm respectively . if the line current on full load is A at speed of 1500 r.m.p. find the no-load speed . **(6)**

Ans: I_{LO} = 4A,V =220 V, R_{sh} = 110 Ω, R_a = 0.2Ω, I_{FL} = 40 A

$I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{220}{110} = 2\ A$

$\therefore \quad I_{a0} = I_{LO} - I_{sh} = 4-2==2A$

$\therefore \quad E_{bo} = V--I_{ao}R_a = 220-2\times0.2=219.6V$

$I_{aFL} = I_{FL}- I_{sh} =40-2=38\ A$ ……… I_{SH} is constant

$\therefore \quad I_{bFL} = V- I_{aFL}\, R_a =220-38\times0.2=212.4V$

$N\propto = \dfrac{E_b}{\phi} \propto E_b$ ……. Φis constant

$\therefore \quad \dfrac{N_o}{N_{Fl}} = \dfrac{E_{bo}}{E_{bFL}}$ i.e $\dfrac{N_o}{1500} = \dfrac{219.6}{212.4}$ ……N_{FL} =1500 r.p.m

$\therefore$ No = 1550.8474 r.p.m …. No load speed.

4) Draw a neat sketch representing the cut section view of a D.C. machine. Explain the important features of different parts involved there on.

Sol.: Construction of a practical D.C. machine:

Whether a d.c. machine is a generator or motor, the basic construction remains the same. The fig. 26 shows the cross- sectional view showing the various parts of four pole, practical d.c. machine.

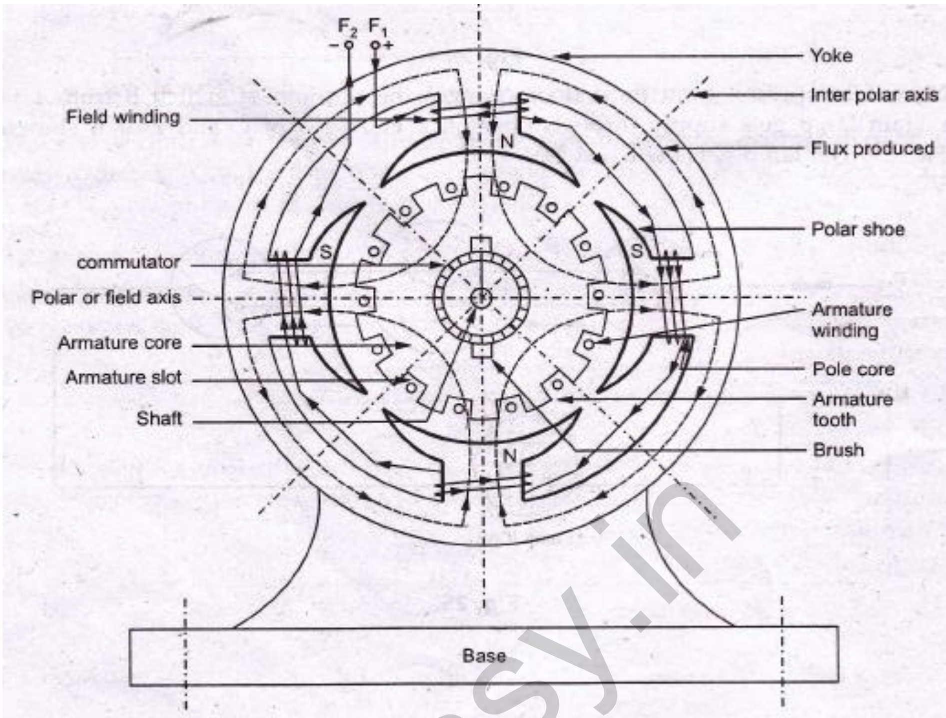


Fig. 26 A cross section of typical d.c. machine

It consists of the following parts:

Yoke:

a) Functions :

i) It serves the purpose of outermost cover of the d.c. machine. So that the insulating materials get protected from harmful atmospheric elements like moisture, dust and various gases like SO_2, acidic fumes etc.

ii) It provides mechanical support to the poles.

iii) It forms a part of the magnetic circuit. It provides a path of low reluctance for magnetic flux. The low reluctance path is important to avoid wastage of power to provide same flux. Large current and hence the power is necessary if the path has high reluctance, to produce the same flux.

b) Choice of material: To provide low reluctance path, it must be made up of some magnetic material. It is prepared by using cast iron because it is cheapest. For large machines rolled steel, cast steel, silicon steel is used which provides high permeability i.e. low reluctance and gives good mechanical strength.

As yoke does not need any machining or good finishing as it rough, casting is the best method of construction of yoke.

Poles:

Each pole is divided into two parts Namely, a) pole core and b) pole shoe

This is shown in fig. 27.

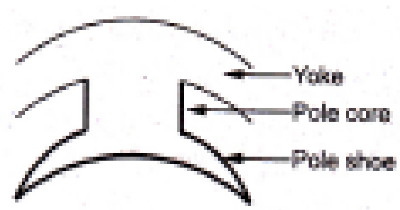


Fig. 27 Pole structure

a) Function of pole core and pole shoe:

i) pole core basically carries a field winding which is necessary to produce the flux.

ii) It directs the flux produced through air gap to armature core, to the next pole.

iii) pole shoe enlarges the area of armature core to come across the flux, which is necessary to produce larger induced e.m.f. to achieve this, pole shoe has given a particular shape.

b) Choice of material : It is made up of magnetic material like cast iron or cast steel.

as it requires a definite shape and size, laminated construction is used. The laminations of required size and shape are stamped together to get a pole, which is then bolted to the yoke.

Field winding [F1-F2] :

The field winding is wound on the pole core with a definite direction.

a) Functions: i) To carry current due to which pole core on which the winding placed behaves as an electromagnet, producing necessary flux.

As it helps in producing the magnetic field i.e. exciting the pole as electromagnet it is called ‘Field winding’ or ‘Exciting winding’.

4) **Explain Choice of material:** As it has to carry current hence obviously made up of some conducting material. So aluminium or copper is the choice. But field coils are required to take any type of shape and bend about pole core and copper has good pliability i.e. it can bend easily. So copper is the proper choice.

Filed winding is divided into various coils called bas field coils. These are connected in series with each other and wound in such a direction around pole cores, such that alternate ‘N’ and ‘S’ poles are formed.

By using right hand thumb rule for current carrying circular conductor, it can be easily determined that how a particular core is going to behave as ‘N’ or ‘S’ for a particular winding direction around it.

Armature:

It is further divided into two parts namely,

I) Armature core and II) Armature winding

I) Armature core is cylindrical in shape mounted on the shaft. It consists of slots on its periphery and the air ducts to permit the air flow through armature which serves cooling purpose.

a) Functions:

i) Armature core provides house for armature winding i.e. armature conductors.

ii) To provide a path of low reluctance to the ma

winding.

b) Choice of material :

A it has to provide a low Reluctance path to the flux, it is made up of magnetic

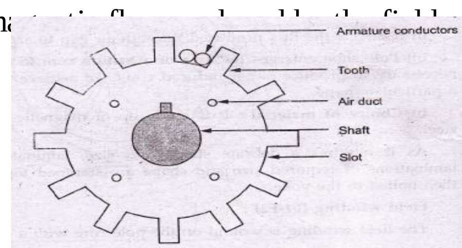


Fig. 28 Single circular lamination of armature core

steel.

It is made up of laminated Construction to keep eddy current Loss as low as possible. A single circular Lamination used for the construction of the armature core is shown in Fig. 28.

II) Armature winding is nothing but the interconnection of the armature conductors, placed in the slots provided on the armature core periphery. When the armature is rotated, in case of generator, magnetic flux gets cut by armature conductors and e.m.f. gets induced in them.

a) Functions:

i) Generation of e.m.f. takes place in the armature winding in case of generators.
ii) To carry the current supplied in case of d.c. motors
iii) To do the useful work in the external circuit.

b) Choice of material: As armature windings carries entire current which depends on external load, it has to be made up of conducting material, which is copper.

Armature winding is generally former wound. The conductors are placed in the armature slots which are lined with tough insulating material.

Commutator:

We have seen earlier that the basic nature of e.m.f induced in the armature conductors is alternating. This needs rectifications in case of d.c. generator which is possible by device called commutator.

a) Functions:

i) To facilitate the collection of current from the armature conductors.
ii) To convert internally developed alternating e.m.f. to unidirectional (d.c.) e.m.f.
iii) To produce unidirectional torque in case of motors.

b) Choice of material: As it collects current from armature, it is also made up of copper segments.

It is cylindrical in shape and is made up of wedge shaped segments of hard drawn, high conductivity copper. Those segments are insulated from each other by thin layer of mica. Each commutator segment is connected to the armature conductor by means of copper lug or strip. This connection is shown in the fig. 29.

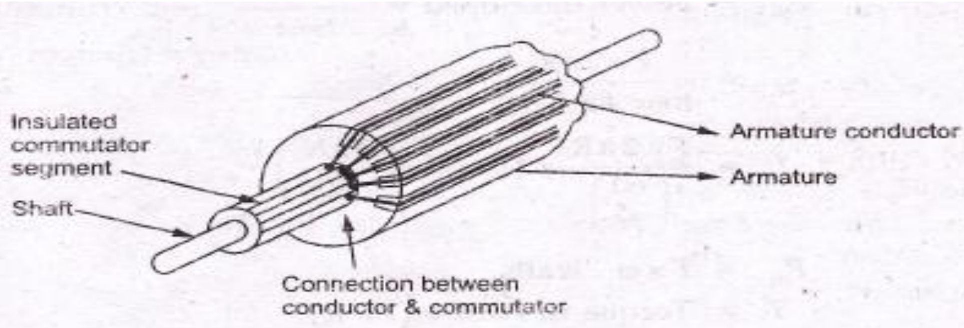


Fig. 29 Commutator

Brushes and brush gear:
Brushes and stationary and resting on the surface of the commutator.

a) Functions:
i) To collect current from commutator and make it available to the stationary external circuit.

b) Choice of material: Brushes are normally made up of soft material like carbon.
Brushes are rectangular in shape. They are housed in brush holders, which are usually of box type. The brushers are made to press on the commutator surface by means of a spring, whose tension can be adjusted with the help of lever. A flexible copper conductor called pig tail is used to connect the brush to the external circuit.

Bearings:
Ball bearings are usually as they are more reliable. For heavy duty machines, roller bearings are preferred.

6) Derive the expression of armature torque developed in a DC motor.

Sol.: Torque equation of D.C. motor:
It is seen that the turning or twisting force about an axis is called torque.

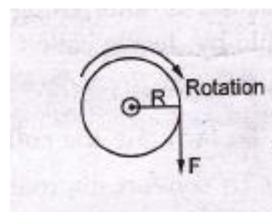


Fig. 30

Consider a wheel of radius R meters, acted upon by a Circumferential force of F Newton's as shown in fig. 30.
The wheel is rotating at a speed of N r.p.m.
Then angular speed of the wheel is,

$$\omega = \frac{2\pi N}{60} \text{ rad/sec}$$

So work done in one revolution is,

W = F× distance travelled in one revolution
= F× 2πR joules

And P_m = power developed = $\frac{\text{work done}}{\text{time}}$

$$= \frac{F \times 2\pi R}{\text{time for 1 rev}}$$

$$= \frac{F \times 2\pi R}{\frac{60}{N}} = (F \times R) \times \left(\frac{2\pi N}{60}\right)$$

$$P_m = T \times \omega$$

where T = Torque in N-m = (F×R)
. ω = angular speed in rad/sec = (2πN/60)

Let Ta be the gross torque developed by the armature of the motor. It is also called armature torque. The gross mechanical power developed in the armature is E_b I_a, as seen from the power equation. So if speed of the motor is N r.p.m. then,

Power in armature = Armature torque × ω

$$E_b I_a = T_a \frac{2\pi N}{60}$$

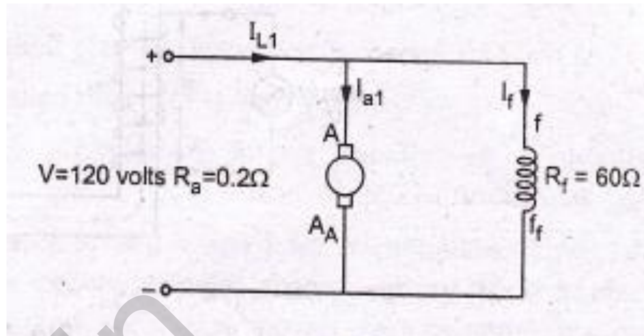


Fig. 31

But E_b in a motor given by, $E_b = \frac{\phi PNZ}{60\ A}$

$$\therefore \quad \frac{\phi PNZ}{60\ A} \times I_a = T_a \times \frac{2\pi N}{60}$$

$$\therefore \quad T_a = \frac{1}{2\pi} \phi I_a \times \frac{PZ}{A}$$

$$\therefore \quad T_a = 0.159\ \phi I_a \frac{PZ}{A} \text{ N-m}$$

This is the torque equation of a d.c.motor.

8) A 120V DC shunt motor has an armature resistance of 60Ω. It runs at 1800 RPM, when it takes full load current of 40A. Find the speed of the motor while it is operating at half the full load, with load terminal voltage remaining same.

Sol.:

$I_f = \frac{120}{60} = 2A'$ $\qquad I_{L1} = 40\ A$

$I_{L1} = I_{a1} + I_f$

$Ia_1 = I_{L1} - I_f = 40 - 2 = 38A$

$E_{b1} = V - I_{a1} R_a = 120 - (38)(0.2)$

$= 120 - 7.6 = 112.4V$

$N_1 = 1800rpm$

Now for shunt machine

$T \propto I_a$ as ϕ remains constant.

For half load

$$T_2 = \frac{1}{2} T_1$$

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$$

$$\frac{\frac{1}{2}T_1}{T_1} = \frac{I_{a2}}{I_{a1}}$$

$I_{a2} = ½\ I_{a1} = ½\ (38) = 19A$

$E_{b2} = V – I_{a2}\ Ra = 120 – (19)(0.2) = 120 – 3.8$

$E_{b2} = 116.2V$

Now, $E_b \propto N$ for Φ constant

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$

$$\frac{116.2}{112.4} = \frac{N_2}{1800}$$

$$N_2 = \frac{116.2}{112.4} \times 1800$$

$N_2 = 1860.85$ rpm

A 4 pole generator with wave wound armature has 51 slots, each having 24 conductors. The flux per pole is 0.01 Weber. At what speed must the armature rotate to give an induced emf of 220 V? What will be the voltage developed if the voltage developed if the winding is lap and the armature rotates at the same speed? [6]

sol.: P = 4, for wave winding A = 2

Number of slots = 51, conductors/slot = 24

$\emptyset = 0.01$ wb

E = 220 volts

Z = Number of slots × conductors/slots

= 51 × 24

= 1224

$$E = \frac{\emptyset ZNP}{60A}$$

$$\therefore \quad N = \frac{E \times 60A}{\emptyset ZP} = \frac{220 \times 60 \times 2}{0.01 \times 1224 \times 4}$$

∴ N = 539.21 rpm

Now speed is same but winding is lap ∴ A = P

$$E = \frac{\emptyset ZNP}{60A} = \frac{\emptyset ZN}{60} = \frac{0.01 \times 1224 \times 539.21}{60}$$

∴ E = 110 volts

A 4 pole, 220 V. lap connected DC shunt motor has 36 slots, each slot containing 16 conductors. It draws a current of 40 A from the supply. The field resistance and armature resistance are 110Ω, 0.1 Ω respectively. The motor develops an output power of 6 kW. The flux per pole is 40 mwb. Calculate

a) the speed

b) the torque developed by the armature and

c) the shaft torque [7]

Sol.: P = 4, V = 220 volts, slots = 36, A = P = 4

Conductors/slot = 16, Z = 36 × 16 = 576

IL = 40 A

$R_{sh} = 110\Omega$ $R_a = 0.1\Omega$

$P_{output} = 6$ KW, $\emptyset = 40\times 10^{-3}$ wb

$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{110} = 2A$

$I_L = I_a + I_{sh}$

$\therefore I_a = I_L - I_{sh}$

$= 40 - 2 = 38A$

$E_b = V - I_aR_a$

$\therefore E_b = 220 - (38)(0.1)$

$= 220 - 3.8$

$\therefore E_b = 216.2$ volts

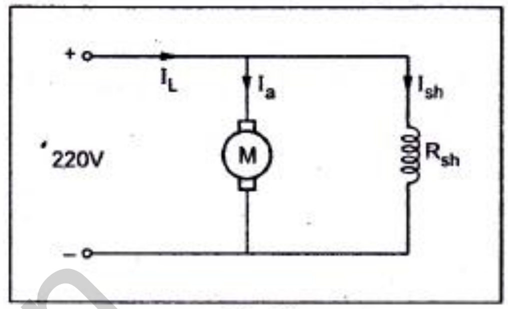


Fig. 9

Now, $E_b = \frac{\emptyset ZNP}{60A}$ $\therefore N = \frac{E_b \times 60A}{\emptyset . Z. P.} = \frac{(216.2)(60)(4)}{(40\times 10^{-3})(576)(4)}$

$\therefore N = 563$ rpm

$T_a = 0.159\ \emptyset\ Z \frac{I_a P}{A} = 0.159\ \emptyset\ ZI_a$

$= (0.159)(40\times 10^{-3})(576)(38)$

$\therefore T_a = 139.20$ N-m

$P_{out} = T_{sh} . \frac{2\pi N}{60}$

$T_{sh} = \frac{P_{out} \times 60}{2\pi N} = \frac{6\times 10^3 \times 60}{2\pi \times 563} = 101.76$ N-m

$\therefore T_{sh} = 101.76$ N-m

A 120V DC shunt motor has an armature resistance of 60Ω. It runs at 1800 RPM, when it takes full load current of 40A. Find the speed of the motor while it is operating at half the full load, with load terminal voltage remaining same.

Sol.:

$I_f = \frac{120}{60} = 2A$, $I_{L1} = 40$ A

$I_{L1} = I_{a1} + I_f$

$Ia_1 = I_{L1} - I_f = 40 - 2 = 38A$

$E_{b1} = V - I_{a1}R_a = 120 - (38)(0.2)$

$= 120 - 7.6 = 112.4V$

$N_1 = 1800rpm$

Now for shunt machine

$T \propto I_a$ as $\emptyset$ remains constant.

For half load

$$T_2 = \frac{1}{2} T_1$$

$$\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}}$$

$$\frac{\frac{1}{2}T_1}{T_1} = \frac{I_{a2}}{I_{a1}}$$

$I_{a2} = ½ I_{a1} = ½ (38) = 19A$

$E_{b2} = V - I_{a2}$ Ra $= 120 - (19)(0.2) = 120 - 3.8$

$E_{b2} = 116.2V$

Now, $E_b \propto N$ for

Φ constant

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$

$$\frac{116.2}{112.4} = \frac{N_2}{1800}$$

$$N_2 = 116.\frac{2}{112}.4 \times 1800$$

$N_2 = 1860.85$ rpm

Transformers: Principle of operation and construction of single-phase transformers (core and shell types). emf equation, losses, efficiency and voltage regulation (Open Circuit and Short circuit tests, equivalent circuit and phasor diagrams are excluded). Illustrative problems on emf equation and efficiency only.

UNIT-6

TRANSFORMERS

TRANSFORMER is a static device which transfer electric energy from one electric circuit to another at any desired voltage with out any change in frequency.

PRINCIPLE:- A transformer works on the principle of mutual induction. "Whenever a change in current takes place in a coil there will be an induced emf in the other coil wound over the same magnetic core". This is the principle of mutual induction by which the two coils are said to be coupled with each other.

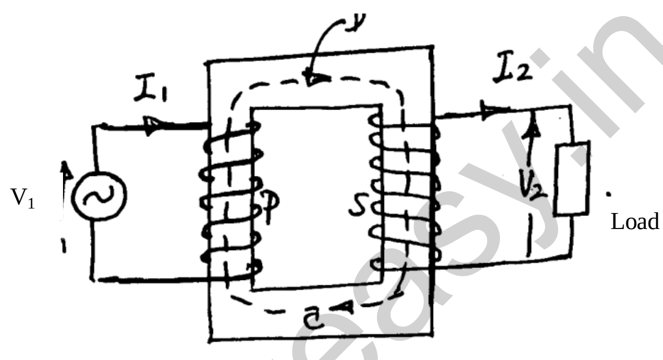


Fig.1

The fig1 shows the general arrangement of a transformer. C is the iron core made of laminated sheets of about 0.35mm thick insulated from one another by varnish or thin paper. The purpose of laminating the core is to reduce the power loss due to eddy currents induced by the alternating magnetic flux. The vertical portions of the core are called limbs and the top and bottom portions are called the yokes. Coils P and S are wound on the limbs. Coil P is connected to the supply and therefore called as the primary, coil S is connected to the load and is called as the secondary.

An alternating voltage applied to P drives an alternating current though P and this current produces an alternating flux in the iron core, the mean path of the flux is represented by the dotted line D. This flux links with the coil S and thereby induces an emf in S.

TYPES AND CONSTRUCTION OF TRANSFORMERS

There are two basic circuits in a transformer

1) **Magnetic circuit** 2) **Electric circuit**

The core forms the magnetic circuit and the electric circuit consists of two windings primary and secondary and is made of pure copper. There are two types of single phase transformers.

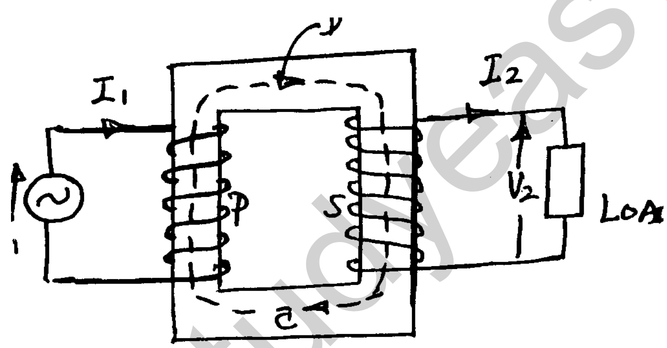
a) CORE TYPE **b) SHELL TYPE**

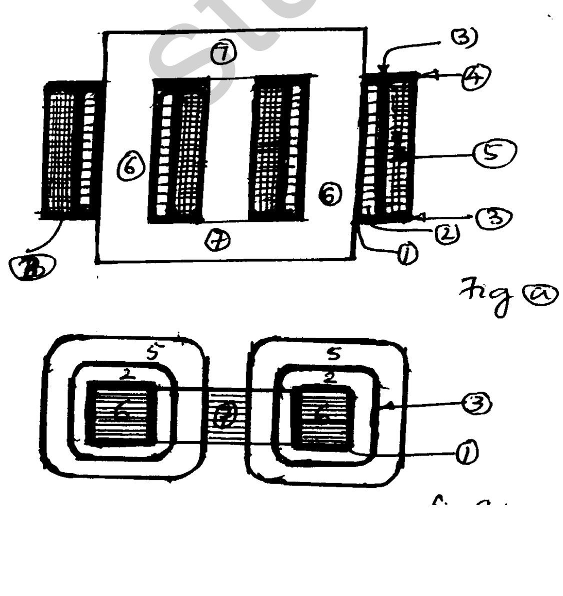
Figs (a) and (b) shows the details of the elevation and plan of a core type transformer. The limbs are wound with half the L.V. and half the H.V. windings with proper insulation between them. The whole assembly taken inside a steel tank filled with oil for the purpose of insulation and cooling.

CORE TYPE TRANSFORMER.

In the core type the core is surrounded by the coils but in the shell type the core is on the either side of the coils. There are three limbs and the central limb is of large cross section than that of outer limbs, and both the LV and HV windings are wound on the central limb and the outer limb is only for providing the return path for the flux.
The windings are of concentric type (i.e. LV on which the HV windings) or Sandwich type.

The core is made of very thin laminations of high grade silicon steel material to reduce the eddy current loss and Hysterisis losses in the core.

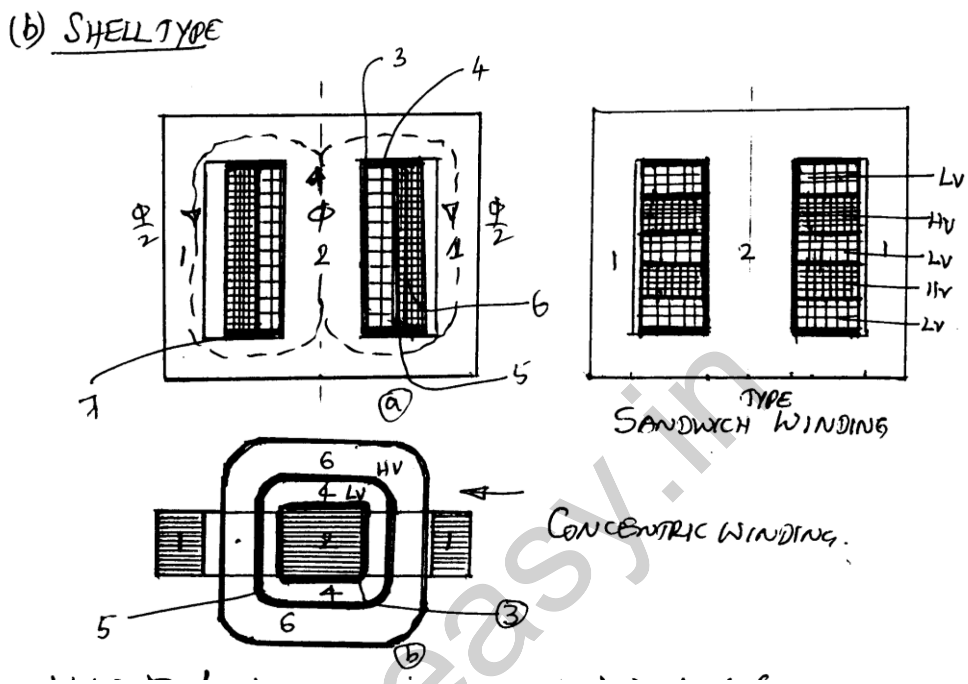




1- Insulation between L V windin
and the core.
2- L V winding.
3- Insulation between L V and H V
winding. 4- End
the yoke.
5- H V winding.
6- Limbs.
7- Yoke.

FIG. (b)

SHELL TYPE TRANSFORMER.



1, 1 – Outer limbs, 4 – L V winding
2 – Central limb, 5 – Insulation between L V and H V windings.
3 – Insulation between L V and Core. 6 – H V winding. 7 – End insulation

In the shell type transformers the core is of different type having three limbs with the central limb of larger cross section compared to the two outer limbs and carries both the L V and H V windings wound over each other with proper insulation between them. The entire assembly is immersed in a steel tank filled with oil for the cooling purpose.

EMF EQUATION:

Priciple:- Whenever a coil is subjected to alternating flux, there will be an induced emf in it and is called the statically induced emf $e = \frac{Nd\phi}{dt}$

Let N_1, N_2 be the no. of turns of the primary and secondary windings, E_1, E_2 the induced emf in the primary and secondary coils. ϕ be the flux which is sinusoidal f be the frequency in Hz

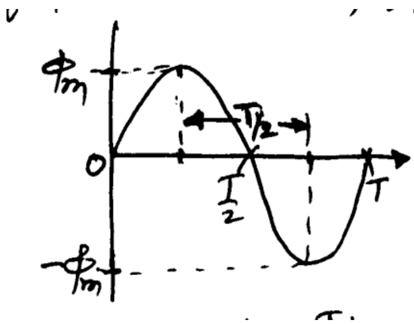


Figure showing the sinusoidally varying flux of peak value Φ_m.

Whenever a coil of **N** no- of tunes are linked by a time varying flux ϕ, the average emf induced in this coil is

$$e = \frac{Nd\phi}{dt}$$

As the flux is sinusoidal the change in flux from $+\phi_m$ **to** $-\phi_m$ is $d\phi = 2\phi m$, and this change takes place in a duration dt = T/2 seconds.

The average induced emf in these N number of turns is

$$\mathbf{E_{avg} = N.d\phi/dt = N.2\phi_m/(T/2) = 4\phi_m N/T = 4f\phi_m N \text{ volts (as } f = 1/T)}$$

We know that the Form factor of a pure sine wave $\mathbf{F.F. = E_{rms}/E_{avg} = 1.11}$

Therefore, $\mathbf{E_{rms} = 1.11\ E_{avg}}$.

$$\mathbf{= (1.11)(4f\phi_m N) = 4.44\ f\phi_m N \text{ volts.}}$$

In the primary coil, $\mathbf{N = N_1}$, $\mathbf{E1 = 4.44f\phi_m N_1}$ **volts**

In the secondary coil, $\mathbf{N = N_2}$, $\mathbf{E_2 = 4.44f\phi_m N_2}$ **volts**

TRANSFORMATION RATIO:

It is defined as the ratio of the secondary induced emf to the primary induced emf.
Therefore, $\mathbf{E_1 / E_2 = N_1 / N_2 = K}$
For an ideal (loss free) transformer, the input power is equal to the out put power.
Therefore $\mathbf{E_1 I_1 = E_2 I_2}$, from which, $\mathbf{E_2/E_1 = I_1/I_2}$
Also the induced emf per turn is same for both the primary and secondary turns.
If the value of the transformation ratio **K>1,** then it is a **step up** case.
If the value of the transformation ratio **K<1**, then it is a **step down** case.
If the value of the transformation ratio **K=1**, then it is a **one:one** transformer.

LOSSES AND EFFICIENCY: There a two types of power losses occur in a transformer

1**) Iron loss** **2) Copper loss**

1) Iron Loss: This is the power loss that occurs in the iron part. This loss is due to the alternating frequency of the emf. Iron loss in further classified into two other losses.

a) Eddy current loss **b) Hysterisis loss**

a) EDDY CURRENT LOSS: This power loss is due to the alternating flux linking the core, which will induced an emf in the core called the eddy emf, due to which a current called the eddy current is being circulated in the core. As there is some resistance in the core with this eddy current circulation converts into heat called the eddy current power loss. Eddy current loss is proportional to the square of the supply frequency.

b) **HYSTERISIS LOSS**: This is the loss in the iron core, due to the magnetic reversal of the flux in the core, which results in the form of heat in the core. This loss is directly proportional to the supply frequency.

Eddy current loss can be minimized by using the core made of thin sheets of silicon steel material, and each lamination is coated with varnish insulation to suppress the path of the eddy currents.

Hysterisis loss can be minimized by using the core material having high permeability.

2) **COPPER LOSS:** This is the power loss that occurs in the primary and secondary coils when the transformer is on load. This power is wasted in the form of heat due to the resistance of the coils. This loss is proportional to the sequence of the load hence it is called the Variable loss where as the Iron loss is called as the Constant loss as the supply voltage and frequency are constants

EFFICENCY: It is the ratio of the out put power to the input power of a transformer

Input = Output + Total losses

= Output + Iron loss + Copper loss

Efficiency =

$$\eta = \frac{outputpower}{outputpower + Ironloss + copperloss}$$

$$= \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + W_{eron} + W_{copper}}$$

Where, V_2 is the secondary (out put) voltage, I_2 is the secondary (out put) current and cos**Φ** is the power factor of the load.

The transformers are normally specified with their ratings as KVA, Therefore,

$$\text{Efficiency} = \frac{(KVA)(10^3)\cos\Phi}{(KVA)(10^3)\cos\Phi + W_{iron} + W_{copper}}$$

Since the copper loss varies as the square of the load the efficiency of the transformer at any desired load x is given by

$$\text{Efficiency} = \frac{x.(KVA)(10^3)\cos\Phi}{x.(KVA)(10^3)\cos\Phi + W_{iron} + (x)^2 W_{copper}}$$

where W_{copper} is the copper loss at full load

$W_{copper} = I^2R$ watts

CONDITION FOR MAXIMUM EFFICIENCY:

In general for the efficiency to be maximum for any device the losses must be minimum. Between the iron and copper losses the iron loss is the fixed loss and the copper loss is the variable loss. When these two losses are equal and also minimum the efficiency will be maximum.

Therefore the condition for maximum efficiency in a transformer is

Iron loss = Copper loss (which ever is minimum)

VOLTAGE REGULATION:

The voltage regulation of a transformer is defined as the change in the secondary terminal voltage between no load and full load at a specified power factor expressed as a percentage of the full load terminal voltage.

$$\text{(percentage voltage regulation)} = \frac{\text{(No load secondary voltage)} - \text{(Full load secondary voltage)}}{\text{(Full load secondary voltage)}} \times 100$$

Voltage regulation is a measure of the change in the terminal voltage of a transformer between No load and Full load. A good transformer has least value of the regulation of the order of $\pm5\%$

A 600 KVA transformer has an efficiency of 92% at full load, uni8ty p.f. and at half load,

Sol. : S = 600 KVA, % η = 92% on full load and half load both

On full load, $\% \eta = \frac{(\text{VA rating})\cos\emptyset_2}{(\text{VA rating})\cos\emptyset_2 + P_i + (P_{cu})F.L.} \times 100$

$0.92 = \frac{600 \times 10^3 \times 1}{600 \times 10^3 \times P_i + (P_{cu})F.L.}$

$P_i + (P_{cu})F.L. = 52173.91$

………… (1)

On half load, $n = \frac{1}{2}$ and

$(P_{cu})H.L. = n^2(P_{cu})F.L. = \frac{1}{4}(P_{cu})F.L.$

$0.92 = \frac{\frac{1}{2} \times 600 \times 10^3 \times 0.9}{\frac{1}{2} \times 600 \times 10^3 \times 0.9 + P_i + \frac{1}{4}(P_{cu})F.L.}$

$P_i + 0.25(P_{cu})F.L. = 23478.261$ ………….. (2)

Subtracting (2) from (1),

$0.75(P_{cu})F.L.$ = 28695.64

$(P_{cu})F.L$ = 38260.86 watts

and P_i = 13913.04 watts

Now n = 0.75 i.e., 75% of full load and $\cos\Phi_2 = 0.9$

(P_{cu}) new = $n^2 (P_{cu})$ F.L. = $(0.75)^2 \times (P_{cu})F.L$.

$\% \eta = \frac{n(\text{VA rating})\cos\emptyset_2}{n(\text{VA rating})\cos\emptyset_2 + P_i + (P_{cu})new} \times 100$

$= \frac{0.75 \times 600 \times 10^3 \times 0.9}{0.75 \times 600 \times 10^3 \times 0.9 + 13913.04 + (0.75)^2 \times 38260.86} \times 100$

= 91.95%

transformer has 80 turns on the secondary.

Calculate :

i) The rated primary and secondary currents

ii) The number of primary turns

iii) The maximum value of flux

iv) Voltage induced per turn.

Sol: KVA rating = 250

V_1 = 11000 volts

$V_2 = 415$ volts

$N_2 = 80$

$\frac{N_2}{N_1} = \frac{V_2}{V_1}$

$N_1 = \frac{N_2}{\left(\frac{V_2}{V_1}\right)} = N2\left(\frac{V_1}{V_2}\right) = 80\left(\frac{11000}{415}\right) = 2120$

$N_1 = 2120$

$KVA = V_1 I_1 = V_2 I_2$

$I_1 = \frac{KVA}{V_1} = \frac{250 \times 10^3}{11000} = 22.72A$

$I2 = \frac{KVA}{V_2} = \frac{250 \times 10^3}{415} = 602.40A$

Neglecting drops in primary,

$V1 = E1 = 11000$

$E1 = 4.44f\,\phi_m \times N1$

$\therefore \quad \frac{11000}{4.44 \times 50 \times 2120} = \phi_m$

$\therefore \quad \phi_m = 0.023$ Wb = 23 mWb

Voltage induced per turn = $\frac{E_1}{N_1} = \frac{11000}{2120} = 5.1886$ volts.

) In a 25 kVA, 2000/200 V Transformer, the iron and copper losses are 350 watts and 400 watts respectively, calculate the efficiency at U.P.F. at half and 3/4th full load.

Sol.: S = 25 k V A

$P_i = 350$ W, $(P_{cu})FL = 400$ W

i) At half load, $\cos\emptyset = 1$

$$\frac{(P_{cu})H.L.}{(P_{cu})F.L.} = \left[\frac{I_{HL}}{I_{FL}}\right]^2 = \left[\frac{\frac{1}{2}I_{FL}}{I_{FL}}\right]^2 = \frac{1}{4}$$

$$(P_{cu})_{HL} = \frac{1}{4} \times 400 = 100 \text{ W}$$

% Efficiency = $\frac{\text{kVA rating} \times \cos\emptyset}{\text{kVA rating} \times \cos\emptyset + P_i + P_{cu}} \times 100$

$$\frac{\frac{1}{2} \times 25 \times 1 \times 10^3}{\frac{1}{2} \times 25 \times 1 \times 10^3 + 350 + 100} \times 100$$

$$= \frac{12.5 \times 10^3}{12.5 \times IC^3 + 450} \times 100$$

$\therefore$ % Efficiency = 96.525

ii) At $\frac{3}{4}$ th load, $\cos\emptyset = 1$

$$\frac{(P_{cu})\frac{3}{4}\text{load}}{(P_{cu})F.L.} = \left(\frac{I_{\frac{3}{4}}}{I_{FL}}\right)^2$$

$$(P_{cu})\frac{3}{4}\text{load} = \left(\frac{3}{4}\right)^2 \times P_{cu\,FL} = \frac{9}{16} \times 400 = 225\ W$$

$$\%\ \text{Efficiency} = \frac{\text{kVA rating} \times \cos\emptyset}{\text{kVA rating} \times \cos\emptyset + P_i + P_{cu}} \times 100$$

$$\frac{\frac{3}{4} \times 25 \times 10^3 \times 1}{\frac{3}{4} \times 25 \times 10^3 + 350 + 225} \times 100$$

$\therefore$ % Efficiency = 97.02%

A 600 KVA transformer has an efficiency of 92% at full load, uni8ty p.f. and at half load, 0.9 p.f. determine its efficiency of 75% of full load and 0.9 p.f.

Sol. : S = 600 KVA, % η = 92% on full load and half load both

On full load, % η $= \frac{(\text{VA rating})\cos\emptyset_2}{(\text{VA rating})\cos\emptyset_2 + P_i + (P_{cu})F.L.} \times 100$

$$0.92 = \frac{600 \times 10^3 \times 1}{600 \times 10^3 \times P_i + (P_{cu})F.L.}$$

$$P_i + (P_{cu})F.L. = 52173.91$$

………… (1)

On half load, $n = \frac{1}{2}$ and

$$(P_{cu})H.L. = n^2 (P_{cu})F.L. = \frac{1}{4}(P_{cu})F.L.$$

$$0.92 = \frac{\frac{1}{2} \times 600 \times 10^3 \times 0.9}{\frac{1}{2} \times 600 \times 10^3 \times 0.9 + P_i + \frac{1}{4}(P_{cu})F.L.}$$

$$P_i + 0.25(P_{cu})F.L. = 23478.261 \qquad \text{………….. (2)}$$

Subtracting (2) from (1),

$0.75(P_{cu})F.L.$ = 28695.64

$(P_{cu})F.L$ = 38260.86 watts

and P_i = 13913.04 watts

Now n = 0.75 i.e., 75% of full load and $\cos\Phi_2 = 0.9$

(P_{cu}) new $= n^2 (P_{cu})$ F.L. $= (0.75)^2 \times (P_{cu})$F.L .

$$\% \eta = \frac{n(\text{VA rating})\cos\varnothing_2}{n(\text{VA rating})\cos\varnothing_2 + P_i + (P_{cu})new} \times 100$$

$$= \frac{0.75 \times 600 \times 10^3 \times 0.9}{0.75 \times 600 \times 10^3 \times 0.9 + 13913.04 + (0.75)^2 \times 38260.86} \times 100$$

= 91.95%

∴

∴

RECOMMENDED QUESTIONS

1. The required No load ratio of a single phase 50Hz, core type transformer is 6000/150v. Find the number of turns per limb on the high and low voltage sides if the flux is to be 0.06 wb.

2. A 125 KVA, single phase transformer has a primary voltage of 2000 v at 60 Hz. If the number of turns of the primary is 182 and secondary has 40 turns find (i) the secondary no load voltage (ii Flux in the core (iii) the full load primary and secondary currents.

3. A 600 KVA, single phase transformer has an efficiency of 92% both at full load and half full load at unity power factor. Determine the efficiency at 75% of full load and 0.9 power factor.
4. A 50KVA transformer has an efficiency of 98% at full load, 0.8 p.f. and an efficiency of 96.9% at 1/4th full load, u.p.f. Determine the iron loss and full load copper loss.

5. In a 50KVA, 2000/200 v single phase transformer the iron loss and full load copper losses area 700 w and 1000 W respectively. Calculate the efficiency of the transformer on full load and at half full load. The p.f. of the load is 0.75 lag.

6. A 5 KVA, 200/100 v, single phase, 50Hz transformer has a rated secondary voltage at full load. When the load is removed the secondary voltage is found to be 110 v. Determine the percentage voltage regulation.

7. A single phase transformer has 400 primary and 1000 secondary turns. The net cross sectional area of the core is 60 cm^2. If the primary winding is connected to a 50Hz supply at 500v, calculate (i) the peak value of the flux density in the core (ii) the voltage induced in the secondary winding.

SOLUTION TO QUESTION BANK

a) With neat sketches explain the constructional details of core type and shell type transformers.

Sol: Core type transformer :

It has a single magnetic circuit. The core is rectangular having two limds. The Winding encircles the core. The coils used are of cylindrical type. As mentioned earlier, the coils are wound in helical layers with different layers insulated from each other by paper or mica. Both the coils are placed on both the limbs. The low voltage coil is placed inside near the core while high voltage coil surrounds the low voltage coil core is made up of large number of thin laminations.

As the windings are uniformly distributed over the two limbs the natural cooling is more effective. The coils can be easily removed by removing the laminations of the top yoke, for maintenance.

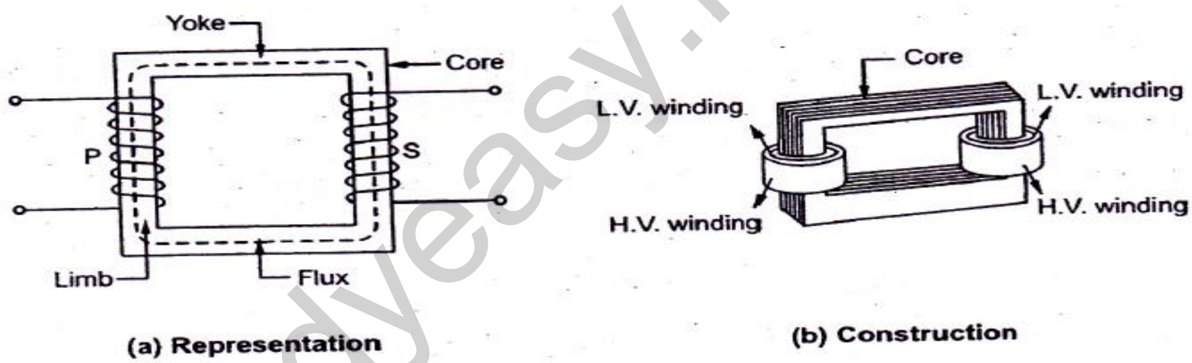


Fig. 12 Core type transformer

The fig.12(a) shows the schematic representation of the core type transformer while the Fig.12(b) shows the view of actual construction of the core type transformer.

2) **Shell type transformer:**

It has a double magnetic circuit. The core has three limds. Both the windings are placed on the central limb. The core encircles most part of the windings. The coils used are generally multilayer disc type or sandwitch coils. As mentioned earlier, each high voltage coil is in between two low voltage coils and low voltage coils are nearest to top and bottom of the yokes.

The core is laminated. While arranging the laminations of the core, the core is taken that all the joints at alternate layers are staggered. This is done to avoid narrow air gap at the joint, right through the cross-section of the core. Such joints are called over lapped or imbricated joints. Generally for very high voltage transformers, the shell type construction is preferred. As the windings are surrounded by the core, the natural cooling does not exist. For removing any winding for maintenance, large number of laminations are required to be removed.

The Fig.13(a) shows the schematic representation while the Fig.13(b) shows the outaway view of the construction of the shell type transformer.

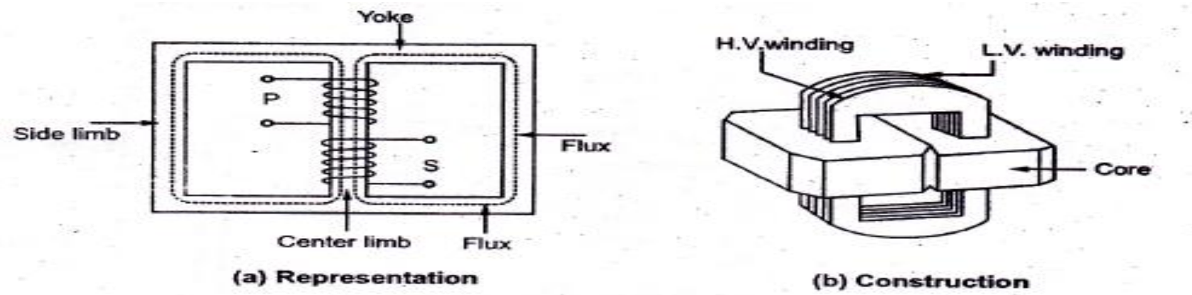


Fig. 13 Shell type transformer

3) A 250 KVA, 11000/415V, 50 Hz single phase transformer has 80 turns on the secondary.

Calculate :

v) The rated primary and secondary currents
vi) The number of primary turns
vii) The maximum value of flux
viii) Voltage induced per turn.

Sol: KVA rating = 250

$V_1 = 11000$ volts

$V_2 = 415$ volts

$N_2 = 80$

$$\frac{N_2}{N_1} = \frac{V_2}{V_1}$$

$$N_1 = \frac{N_2}{\left(\frac{V_2}{V_1}\right)} = N2\left(\frac{V_1}{V_2}\right) = 80\left(\frac{11000}{415}\right) = 2120$$

$N_1 = 2120$

$KVA = V_1 I_1 = V_2 I_2$

$$I_1 = \frac{KVA}{V_1} = \frac{250\times10^3}{11000} = 22.72A$$

$$I2 = \frac{KVA}{V_2} = \frac{250\times10^3}{415} = 602.40A$$

Neglecting drops in primary,

$V1 = E1 = 11000$

$E1 = 4.44f\, \phi_m \times N1$

$$\therefore \quad \frac{11000}{4.44\times50\times2120} = \phi_m$$

$\therefore \quad \phi_m = 0.023$ Wb = 23 mWb

Voltage induced per turn = $\frac{E_1}{N_1} = \frac{11000}{2120} = 5.1886$ volts.

4) Develop an expression for the efficiency of a single phase transformer and obtain the condition for maximum efficiency.

Sol.: Efficiency of transformer: Explain the principle of operation of a single phase transformer and derive its EMF equation.

Sol.: principle of working: A transformer works on the principle of electromagnetic induction and mutual induction between the two coils. To understand this, consider the elementary transformer shown in fig. 32.

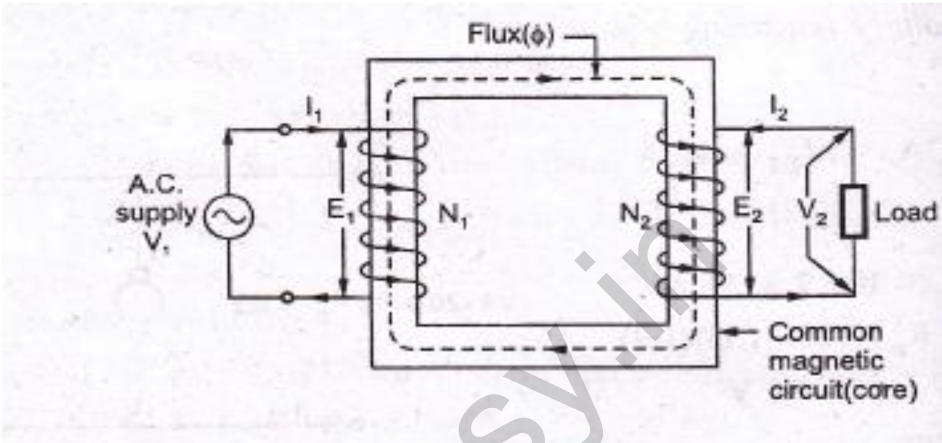


Fig. 32

It consists of two separate electrical windings which linked through common magnetic circuit. electrically windings are isolated from each other.

The coil in which electrical energy is fed is called primary winding (P) while the other from which electrical energy is drawn out is called secondary winding (S). The primary winding has N_1 number of turns while secondary windings has N_2 number of turns.

When primary winding is excited by alternating voltage say V_1, it circulate alternating current through it. This current produces an alternating flux Φ which completes its path through the common magnetic core as shown in fig. 32. This flux links with both the windings.

Because of this it produces self induced e.m.f. E_1 in the primary winding while due to mutual induction i.e. due to flux produced by primary linking with secondary, it produces induced e.m.f. E_2 in secondary winding.

These e.m.f.s are, $$E_1 = -N_1 \frac{d\phi}{dt}$$

$$E_2 = -N_2 \frac{d\phi}{dt}$$

Taking ratio of the above expressions,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$ The frequency of two e.m.f.s is same.

If now secondary circuit is closed through the load, the mutually induced e.m.f. in the secondary winding circulates current through the load. Thus electrical energy is transferred from primary to secondary with the help of magnetic core.

A voltage V_2 appears across the load.

Hence V_1 is the supply voltage, while V_2 is the secondary voltage when load is connected then,

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \text{i.e.} \quad V_2 = V_1\left(\frac{N_2}{N_1}\right)$$

$$K = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \text{transformation ratio}$$

If $k > 1$ then $v_2 > v_1$ transformer is called step up transformer.
If $k < 1$ then $v_2 < v_1$ transformer is called step down transformer
If $k = 1$ then $v_2 = v_1$ transformer is called one to one transformer or isolation transformer.

The current flowing through primary is I_1 and when load is connected current I_2 flows through secondary winding. The powder transfer from primary to secondary remains same. Assuming both primary and secondary power factors same we can write,

Power input to primary = power output from secondary

$$V_1 I_1 = V_2 I_2$$

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{E_1}{E_2}$$

E.M.F. equation of a transformer:

Primary winding is exicted by a voltage which is alternating in nature. This circulates current through primary which is also alternating and hence the flux produced is also sinusoidal in nature.

Please refer Fig. 33 on next page.

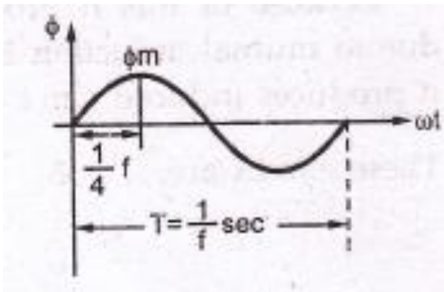


Fig. 33

$\Phi = \Phi_m \sin\omega t$ Wb
Φ_m = Maximum flux
N_1 = Primary turns
N_2 = secondary turns
F = frequency of supply voltage Hz.
E_1 = RMS value of primary induced e.m.f.
E_2 = RMS value of secondary induced e.m.f.

From Faraday's law of electromagnetic induction,
Average e.m.f. induced in each turn = Average rate of change of flux

$= \frac{d\phi}{dt}$ (considering magnitude only)

To find average rate of change of flux, consider 1/4th cycle of flux as shown in the fig. 33. Complete cycle gets completed in 1/f seconds so for 1/4th cycle time required is 1/4f seconds. The flux reaches to Φ_m starting from zero during this time.

$$\frac{d\phi}{dt} = \frac{\Phi m}{1/4f} = 4f\Phi_m \quad \text{Wb/ sec}$$

e purn turn = $4f\Phi_m$ volts

since Φ is varying sinusoidally with time, the induced e.m.f. in each turn of both the windings is also sinusoidal in nature.

For sinusoidal quantity , form factor = $\frac{RMS}{average} = 1.11$

RMS value of induced e.m.f. per unit turn = 1.11×e = $4.44f\Phi_m$

RMS value of induced e.m.f. in primary winding = $4.44\ f\Phi_m\ N_1$ volts

RMS value of induced e.m.f. in secondary winding = $4.44\ f\Phi_m\ N_2$ volts

Due to losses power output of transformer is always less than power input.

Power output = power input – losses

Hence efficiency is defined as ratio of output power to input power both being in watts or kilowatts.

$$\eta = \frac{output}{input} = \frac{output}{output + losses}$$

$$= \frac{output}{output + ironloss + copper\ losses}$$

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + P_{cu}}$$

where P_i = iron loss and $\cos\phi_2$ is load p.f.

P_{cu} = copper loss

$$\eta = \frac{(\text{Load } V_A)\cos\phi_2}{(\text{Load } V_A)\cos\phi_2 + P_1 + I_2^2 R_{2e}} \quad \text{for full load}$$

But $V_2I_2 = V_1I_1$= VA rating of transformer

$$\eta = \frac{(\text{VA rating})\cos\phi_2}{(\text{VA rating})\cos\phi_2 + P_i + P_{cu}}$$

The $\cos\phi_2$ is the power factor of the load. If load is not full load, then load current I_2 reduces by same propotion i.e. for half load half of full load, for 1/4th load 1/4th of full load and so on. Thus output VA available which is V_2I_2 also reduces by same proportion. While copper losses reduces propotional to square of current.

Thus if 'n' is the fraction of full load applied to the transformer then efficiency is given by,

$$\% \eta = \frac{n(\text{VA rating})\cos\phi_2}{n(\text{VA rating})\cos\phi_2 + P_i + n^2 \times (P_{cu})\text{F.L.}} \times 100$$

Where n < 1 which is fraction of full load

6) Condition for maximum efficiency:

When transformer works on a constant input voltage and frequency and with a variable load, its efficiency varies. Let us find out load at which efficiency is maximum and the condition for the maximum efficiency.

The efficiency depends on the load current I_2. Assume the power factor of load constant and secondary voltage V_2 substantially constant. The for maximum efficiency,

$$\frac{d\eta}{dI_2} = 0$$

$$\frac{d}{dI_2}\left[\frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}\right] = 0 \qquad \text{as} \quad P_{cu} = I_2^2 R_{2e}$$

i.e. $(V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e})(V_2 \cos\phi_2) - (V_2 I_2 \cos\phi_2)(V_2 \cos\phi_2 + 2I_2 R_{2e}) = 0$

$V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e} - V_2 \cos\phi_2 . I_2 - 2I_2^2 R_{2e} = 0$ (cancelling $V_2 \cos \phi_2$)

$$P_i - I_2^2 R_{2e} = 0$$

$$P_i = I_2^2 R_{2e} = P_{cu}$$

Thus for maximum efficiency,

Iron loss = copper loss

Now, $I_2^2 R_{2e} = P_i$

$$I_2 = \sqrt{\frac{P_i}{R_{2e}}} \text{ Amp for } \eta_{max}$$

This is the load current at which efficiency is maximum. This is denoted as I_2 max. let us find out KVA at which maximum efficiency occurs:

Let (p_{cu}) F.L. $\propto$ (full load KVA)^2 as $P_{cu} \propto I^2$

At η_{max}, $P_i = P_{cu}$

Let x be the KVA at which efficiency is maximum.

$$P_{cu} \text{ at } \eta_{max} \propto x^2$$

but $P_{cu} = P_i$ hence, $P_i \propto x^2$ at η_{max}

taking ratio of (1) and (2) ,

$$\frac{(P_{cu})\text{F.L}}{P_i} = \left(\frac{\text{full load kVA}}{x}\right)^2$$

$$x = \text{full load kVA} \times \sqrt{\frac{P_i}{(P_{cu})\text{F.L}}}$$

Thus x is the new kVA at which efficiency is maximum.

Now at η_{max}, $P_i = P_{cu}$

$$\% \eta_{max} = \frac{x \cos\phi_2}{x \cos\phi_2 + 2P_i} \times 100$$

Now the current at η_{max}, is $I_{2\ max}$ hence η_{max} m cxanalso be expressed as,

$$\% \eta_{max} = \frac{V_2 I_{2max} \cos\phi_2}{V_2 I_{2max} \cos\phi_2 + 2P_i} \times 100$$

8) A 600 KVA transformer has an efficiency of 92% at full load, uni8ty p.f. and at half load, 0.9 p.f. determine its efficiency of 75% of full load and 0.9 p.f.

Sol. : S = 600 KVA, % η = 92% on full load and half load both

$$\eta = \frac{(\text{VA rating}) \cos\phi_2}{(\text{VA rating}) \cos\phi_2 + P_i + (P_{cu}) F.L.} \times 100$$

$$0.92 = \frac{600 \times 10^3 \times 1}{600 \times 10^3 \times P_i + (P_{cu}) F.L.}$$

$$P_i + (P_{cu}) F.L. = 52173.91$$

………… (1)

On half load, $n = \frac{1}{2}$ and

$$(P_{cu}) H.L. = n^2 (P_{cu}) F.L. = \frac{1}{4} (P_{cu}) F.L.$$

$$0.92 = \frac{\frac{1}{2} \times 600 \times 10^3 \times 0.9}{\frac{1}{2} \times 600 \times 10^3 \times 0.9 + P_i + \frac{1}{4}(P_{cu}) F.L.}$$

$$P_i + 0.25 (P_{cu}) F.L. = 23478.261 \qquad \text{………….. (2)}$$

Subtracting (2) from (1),

$0.75 (P_{cu}) F.L. = 28695.64$

$(P_{cu}) F.L = 38260.86$ watts

and $P_i = 13913.04$ watts

Now $n = 0.75$ i.e., 75% of full load and $\cos\Phi_2 = 0.9$

(P_{cu}) new $= n^2 (P_{cu})$ F.L. $= (0.75)^2 \times (P_{cu})$ F.L.

$$\% \eta = \frac{n(\text{VA rating}) \cos\phi_2}{n(\text{VA rating}) \cos\phi_2 + P_i + (P_{cu}) \text{new}} \times 100$$

$$= \frac{0.75 \times 600 \times 10^3 \times 0.9}{0.75 \times 600 \times 10^3 \times 0.9 + 13913.04 + (0.75)^2 \times 38260.86} \times 100$$

= 91.95%

d) A 25 kVA transformer has an efficiency of 94% at full load unity p.f. and at half full load, 0.9 p.f. determine the iron loss and full load copper loss (6)

Ans. : η_{FL} = 94% at cos Φ = 1, n_{HL} = 94% at cos Φ= 0.9

at full load, $P_{cu} = (P_{cu})_{fl}$

At half load $P_{cu} = (P_{cu})_{HL} = n^2$ (pcu)FL ...n = 0.5

$$\therefore \quad \% \eta FL = \frac{VA \cos\phi}{VA \cos\phi + (Pcu) FL + P_i} \times 100$$

$$\therefore \quad 0.94 = \frac{25 \times 10^3 \times 1}{25 \times 10^3 \times 1 + (Pcu)FL + P_i}$$

$(P_{cu})_{FL} + P_i = 1595.7446$ ...(1)

$$\therefore \quad \%\eta_{FL} = \frac{n \times VA \cos\phi}{n \times VA \cos\phi + n^2 (Pcu) FL + P_i} \times 100 \quad \text{...n = 0.5}$$

$$0.94 = \frac{0.5 \times 25 \times 10^3 \times 0.9}{0.5 \times 25 \times 10^3 \times 0.9 + 0.25 (Pcu)FL + P_i}$$

$0.25(P_{cu})_{FL} + P_i = 718.0851$ ...(2)

Subracting equations (1) and (2),

$0.75(P_{cu})_{FL} = 877.6595$

$\therefore$ $(P_{cu})_{FL} = 1170.2126$ W, $P_i = 425.5319$ W.

c) A 250 KVA, 11000/415V, 50 Hz single phase transformer has 80 turns on the secondary.

Calculate :

i) The rated primary and secondary currents
ii) The number of primary turns
iii) The maximum value of flux
iv) Voltage induced per turn.

Sol: KVA rating = 250

$V_1 = 11000$ volts

$V_2 = 415$ volts

$N_2 = 80$

$$\frac{N_2}{N_1} = \frac{V_2}{V_1}$$

$$N_1 = \frac{N_2}{\left(\frac{V_2}{V_1}\right)} = N2\left(\frac{V_1}{V_2}\right) = 80\left(\frac{11000}{415}\right) = 2120$$

$N_1 = 2120$

$KVA = V_1 I_1 = V_2 I_2$

$$I_1 = \frac{KVA}{V_1} = \frac{250 \times 10^3}{11000} = 22.72A$$

Synchronous Generators: Principle of operation. Types and constructional features. emf equation. Concept of winding factor (excluding derivation of distribution and pitch factors). Illustrative examples on emf. equation.

UNIT-7

THREE PHASE ALTERNATORS.
(SYNCHRONOUS GENERATORS)
(THREE PHASE A.C. GENERATORS).

Electric power is generated using three phase alternators.

Principle: Whenever a coil is rotated in a magnetic field an EMF will be induced in the coil. This is called the dynamically induced EMF.

Alternators are also called as Synchronous Generators due to the reason that under normal conditions the generator is to be rotated at a definite speed called "SYNCHRONOUS SPEED", Ns R.P.M. in order to have a fixed frequency in the output EMF wave.

Ns is related with the frequency as $Ns = 120f / P$, where f is the frequency and P is the total number of poles.

The following table gives the idea of the various synchronous speeds for various numbers of poles for the fixed frequency of 50 Hz.

P	**2**	**4**	**6**	**8**	**10**	**12**	**16**	**……….**
Ns rpm	**3000**	**1500**	**1000**	**750**	**600**	**500**	**375**	**……….**

<u>TYPES AND THEIR CONSTRUCTION</u>:

Their two basic parts in an alternator: (i) Stator, (ii) Rotor.
Stator is the stationary part and Rotor is the revolving part.
There are two possibilities that (i) The armature can be the stator and the field system can be the rotor, and (ii) The armature can be the rotor and the field system be the stator. In practice large alternators are of the first type where in the stator is the armature and the rotor is the field system. And this type is called the "REVOLVING FIELD TYPE".
Revolving field types are preferred due to the following reasons:

(i) More conductors can be easily accommodated and with these high voltage and higher power capacity can be achieved.
(ii) Armature conductors can be easily braced over a rigid frame.
(iii) It is easier to insulate a stationary system.

(iv) Cooling of the conductors will be very effective with proper cooling ducts / vents in the stationary part.
(v) Power can be tapped easily with out any risk from the stationary part through terminal bushings.
(vi) The armature conductors are totally free from any centrifugal force action which tends to drag the conductors out of the slots.

ONSTRUCTION:

Revolving field type alternators are further classified into two types:
(i) Salient pole type, (ii) Non-salient pole type or Cylindrical rotor type.

Figs. (a), (b) and (c) shows the constructional features of the Alternator. Fig. (a) represents the stator, the core of which is made of steel laminations with slots cut in its inner periphery and all the stator stampings are pressed together and are fixed to the stator frame. Three phase windings are accommodated in these slots. These coils are identical to each other and are physically distributed such that they are displaced from each other by 120 degrees as shown in fig. (d).

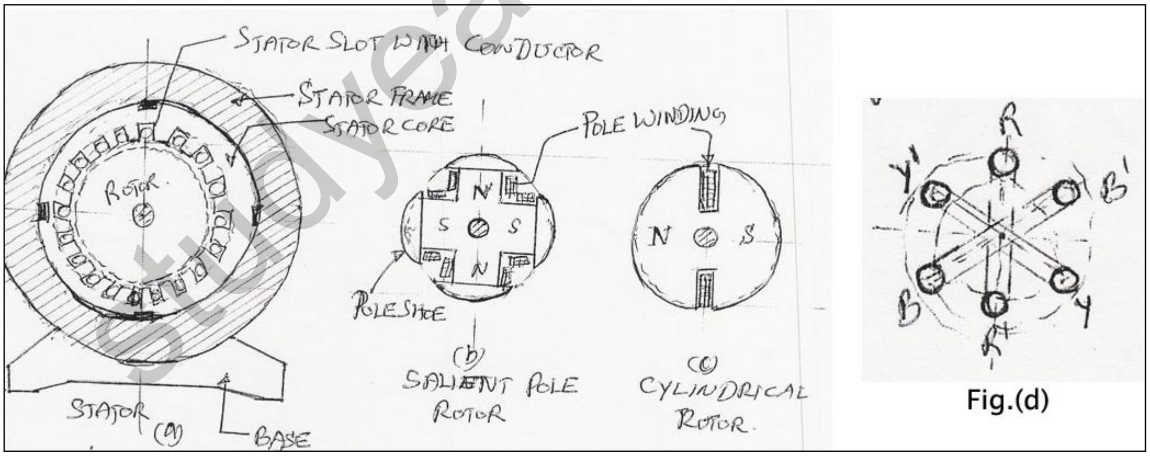
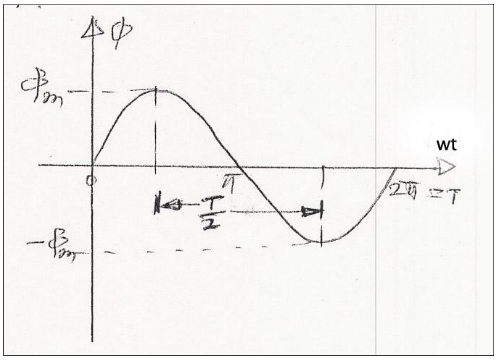


Fig.(d)

Fig. (b) represents the structure of a salient pole rotor where the poles are of projected type and are mounted on a spider and the field or the pole windings are wound over the pole core as shown. This type is preferred where the running speeds are low. Fig.(c) represents the structure of a non-salient pole rotor where the overall structure is like a cylinder having 2 or 4 poles. This type is preferred where the running speeds are very high.

The armature windings in the stator are made of copper and are normally arranged in two layers and are wound for lap or wave depending on the requirements and are usually connected in star with the neutral terminal brought out.

EMF Equation:



Let P be the total number of poles, Ns be the synchronous speed, f be the frequency of the induced EMF and the flux Φ considered to be sinusoidally distributed.
As we know that the induced emf is due to the rate of change of flux cut by coils, the average induced emf in Tph number of turns is

Eavg = Tph dΦ / dt volts.

For a flux change from Φm to Φm is d Φ = 2 Φm in time dt= T / 2 seconds,
The average induced Emf = Tph. 2 Φ m / (T/2) = 4 Tph .f. m volts.
For a sine wave we know that the form factor is of value 1.11= Erms / Eavg.
Therefore, Erms = 1.11.Eavg.
Erms = 4.44 f Φm Tph volts per phase. . . . .(1)

If the armature windings are connected in star the line emf is El = 3 Ephase.
If the armature windings are connected in delta the line emf is the phase emf itself.
Equation (1) represents the theoretical value of the induced em f in each phase but in practice the
Induced emf will be slightly less than the theoretical value due to the following reasons:

(i) The armature windings are distributed throughout the armature in various slots and this is accounted by a factor called the "Distribution factor" Kd and is given by

Kd = (Sin(mα / 2) / mSin(α / 2)), where m is the number of slots per pole peer phase
and α is the slot angle.
α = 180^0 / no. of slots per pole.

(ii) The span of the armature coil is less than a full pitch – This is done deliberately to eliminate some unwanted harmonics in the emf wave, this fact is accounted by a factor called the coil span factor or the pitch factor, Kp and is given by

Kp = Cos (β / 2), where β is the angle by which the coils are short chorded.

The modified Emf equation with these two factors taken into account will be

E = 4.44 Kd.Kp..f Tph volts per phase.

The product of K_d and K_p is called as the winding factor Kw .which is of value around 0.95.

VOLTAGE REGULATION:

The voltage regulation of an alternator is defined as the change in the terminal voltage between no load and full load at a specified power factor, without any change in the speed and excitation.

$$\% \text{ Voltage regulation} = \frac{\text{(No load terminal voltage – Full load terminal voltage)}}{\text{Full load terminal voltage}} \times 100$$

$$\% \text{ Voltage regulation} = \frac{E - V}{V} \times 100$$

The idea of voltage regulation is necessary to judge the performance of an alternator. Lesser the value of the regulation better will be the load sharing capacity at better efficiency.

A 3 phase, 6 pole, star connected alternator has 48 slots and 12 conductors per slot on the armature. If the rotor at 1200RPM and the flux per pole is 0.3 Wb, calculate the e.m.f. induced in the armature. The coils are full pitched and the winding factor is 0.95.

sol. : P = 6, slots = 48, conductors/ slots = 12, Φ = 0.3 wb

$N_s = \frac{120 \times f}{P}$; N_s = 1200 rpm

$f = \frac{PN_S}{120} = \frac{6 \times 1200}{120} = 60\ Hz$

for full pitched winding, $K_c = 1$, $K_d = 0.95$

Total conductors = slots × conductors/slot = 48×12 = 576

$\frac{Z}{3} = \frac{576}{3} = 192$

$Z_{ph} = \frac{Z_{ph}}{2} = \frac{192}{2} = 96$

T_{ph} =

$E_{ph} = 4.44\ K_c\ K_d\ f\ \Phi\ T_{ph}$

$E_{ph} = 4.44 \times 1 \times 0.95 \times 60 \times 0.3 \times 96$

$= 75.9 \times 96$

E_{ph} = 7288.70 volts √3×7288.70

E_{line} = 12.624 Kv

REVIEW PROBLEMS:

1. The stator of a three phase, 8 pole, 750 rpm alternator has 72 slots, each of which having 10 conductors. Calculate the RMS value of the emf per phase if the flux per pole is 0.1 wb and the winding factor is 0.96. Also find the line Emf if the windings are connected in (i) Star, and (ii) Delta.
2. A three phase star connected alternator driven at 900 rpm is required to generate a line voltage of 460 volts at 60 Hz on open circuit. The stator has 2 slots per pole per phase and 4 conductors per slot. Calculate (i) the number of poles, (ii) the useful flux per pole.
3. A 4 pole, three phase, 50 Hz, star connected a.c. generator has 24 stator slots. Find the number of conductors per slot if the flux per pole is 62 mwb and the terminal voltage is 1100 volt. Assume full pitch coils.
4. A 16 pole star connected alternator has 144 slots and 10 conductors per slot. The flux per pole is 30 mwb and the speed is 375 rpm. Find the frequency, the phase and line emf's.
5. Find the number of armature conductors in series per phase required for the armature of a three phase
   50 Hz 10 pole alternator with 90 slots. The winding is star connected to give a line emf of 11 Kv. The flux is 0.16 wb. Also find the voltage regulation if the terminal voltage on full load is 11.2 kv.
6. A three phase 10 pole star connected alternator runs at 600 rpm. It has 120 stator slots with 8 conductors per slot. Determine the phase and line emf's if the flux per pole is 56 mwb.
7. Calculate the phase emf induced in a 4 pole, three phase, 50 Hz star connected alternator with 36 slots and 30 conductors per slot. The flux per pole is 0.05 wb. Given the winding factor as 0.95.

SOLUTION TO QUESTION PAPER

a) Explain the principle of operation of an alternator. Discuss the different types of rotor construction of alternator mentioning their typical advantages and applications.
[8]

Sol. : Working principle of alternator :

The alternators work on the principle of electromagnetic induction. When there is a relative motion between the conductors and the flux, e.m.f. gets induced in the conductors. The d.c. gemerators also work on the same principle. The only difference in practical alternator and a d.c. generator is that in an alternator the conductors are stationary and field is rotating. But for understanding purpose we can always consider relative motion of conductors with respect to the flux produced with respect to the flux produced by the field winding.

Consider a relative motion of a single conductor under the magnetic field produced by two stationary poles. The magnetic axis of the two poles produced by field is vertical, shown dotted in the fig. 34.

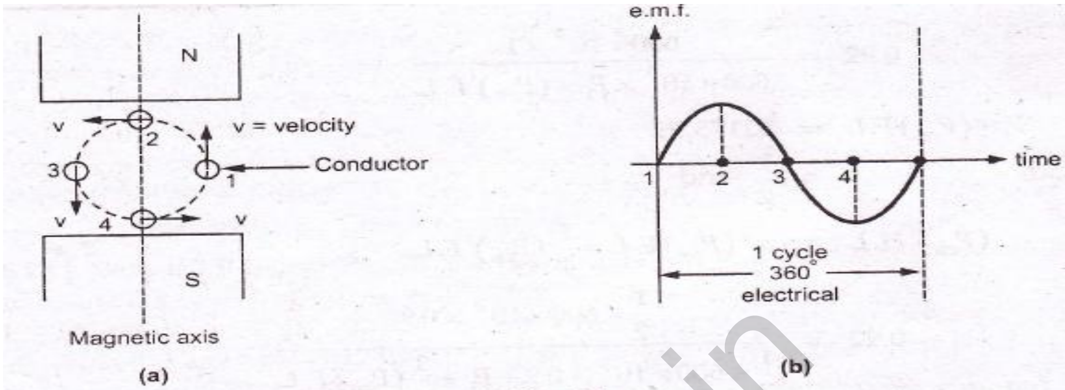


Fig. 34

Let the conductor starts rotating from position 1. At this instant, the entire velocity component is parallel to the flux lines. Hence there is no cutting of flux lines by the conductor. So $\frac{d\phi}{dt}$ at this instant is zero and hence induced e.m.f. in the conductor is also zero.

As the conductor moves from position 1 to position 2, the part of the velocity component becomes perpendicular to the flux lines and propotional to that e.m.f. increases as the conductor moves from position 1 towards 2.

At position 2, the entire velocity component is perpendicular to the flux lines. Hence there exists maximum cutting of the flux lines. And at this instant, the induced e.m.f. in the conductor is at its maximum.

As the position of the conductor changes from 2 towards 3, the velocity component perpendicular to the flux starts decreasing and hence induced e.m.f. magnitude also starts decreasing. At position 3, again the entire velocity component is parallel to the flux lines and hence at this instant induced e.m.f. in the conductor is zero.

As the conductor moves from position 3 towards 4, the velocity component perpendicular to the flux lines again starts increasing. But the direction of velocity component now is opposite to the direction of velocity component existing during the movement of the conductor from position 1 to 2. Hence induced e.m.f. in the conductor increases but in the opposite direction.

At the position 4, it achives maxima in the opposite direction, as the entire velocity component becomes perpendicular to the flux lines.

Again from position 4 to 1, induced e.m.f. decreases and finally at position 1,again becomes zero. This cycle continues as conductor rotates at a certain speed.

So if we plot the magnitudes of the induced e.m.f. against the time, we get alternating nature of the induced e.m.f. as shown in the fig. 34 (b).

This is the working principle of an alternator.

Construction : Most of the alternators prefer rotating filed type of construction. In case of alternators the winding terminology is slightly different than in case of d.c. generators. In alternators the stationary winding is called ‘stator’ while the rotating winding is called ‘rotor’. So most of alternators have stator as armature and rotor as field, in practice. Constructional details of rotating field type of alternator are discussed below.

1) stator : The stator is a stationary armature. This consists of a core and the slots to hold the armature winding similar to the armature of a d.c. generator. The stator core uses a laminated construction. It is built up of special steel stampings insulated from each other with varnish or paper. The laminated construction is basically to keep down eddy currents losses. The entire core is fabricated in a frame made up of steel plates. The core has slots om its periphery for housing the armature conductors. Frame does not carry and flux and serves as the support to the core. Ventilation is maintained with the help of holes cast in the frame. The section of an alternator startor is shown in the fig. 35.

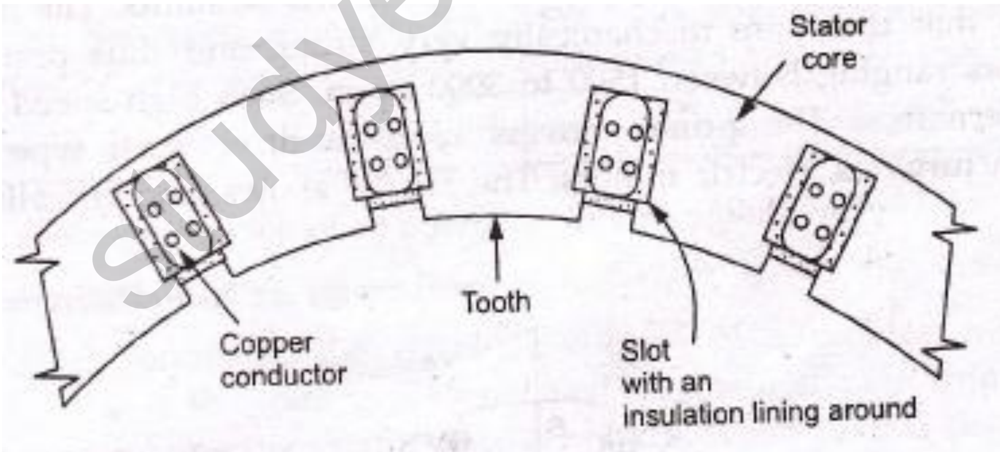


Fig. 35 Section of an alternator stator

2) Rotor :

There are two types of rotors used in alternators which are:
1) salient pole type and 2) smooth cylindrical type.

3) salient pole type :

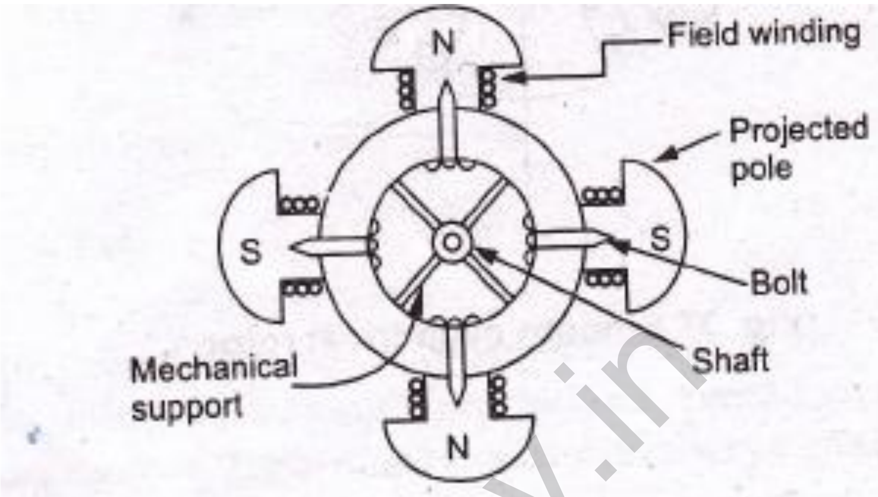


Fig. 36 Salient pole type rotor

This is also called projected pole type as all the poles are projected out from the surface of the rotor. The poles are built up of thick steel laminations. The poles are bolted to the rotor as shown in the fig. 36. The pole face has been given a specific shape as discussed earlier in case of d.c. generators. The field winding is provided on the pole shoe. These rotors have large diameters and small axial lengths. The limiting factor for the size of the rotor is centrifugal force acting on the rotating member of the machine. As mechinal strength of sailent ple type is less, this is preferred for low speed alternators ranging from 125 r.p.m. to 500 r.p.m. the prime movers used to drive such rotor are generally water turbines and I.C. engines.

ii) Smooth cylindrical type :

This is also called non salient type or non – projected pole type of rotor.

The rotor consists of smooth solid steel cylinder, having number of slots to accommodate the field coil. The slots are covered at the top with the help of steel or manganese wedges. The unslotted portion of the cylinder itself act as the poles. The poles are not projecting out and the surface of the rotor is smooth which maintains uniform air gap between and the rotor. These rotors have small diameters and large axial lengths. This is to keep peripheral speed within limits. The main advantage of this type is that are

mechanically very strong and thus preffered for high speed alternators ranging between 1500 to 3000 r.p.m. such high speed alternators are called turboalternators. The prime movers used to drive such type of rotors are generally steam turbines, electric motors. The fig. 37 shows smooth cylindrical type of rotor.

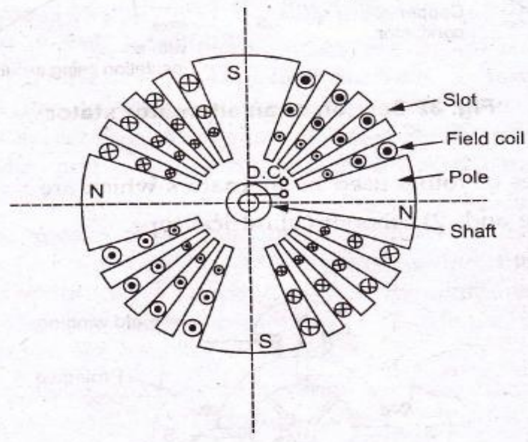


Fig. 37 Smooth cylindrical rotor

Difference between salient and cylindrical type of rotor :

	Salient pole type	Smooth cylindrical type
1	poles are projecting out from the surface.	portion of the cylinder acts as poles hence poles are non projecting.
2	Air gap is non uniform.	Air gap is uniform due to smooth cylindrical periphery.
3	Diameter is high and axial length is small.	Small diameter and large axial length is the feature.
4	Mechanically weak.	Mechanically robust.
5	Preferred for low speed alternators.	Preffered for high speed alternators i.e. for tuboalternators.
6	Prime mover used is water turbines, I.C. engines.	Prime mover used are steam turbines, electric motors.
7	For same size, the rating is smaller than cylindrical type.	For same size, rating is higher than sailent pole type.
8	Separate damper winding is provided	Separate damper winding is not necessary.

b) Explain voltage regulation of an alternator and its significance.

Sol. : Voltage regulation of an alternator :

Under the load condition, the terminal volatage of alternator is less than the induced e.m.f. E_{ph}. So if the load is disconnected, V_{ph} will change from V_{ph} to $E_{ph,}$ if flux and speed is maintained constant. This is because when load is disconnected I_a is zero hence there are no voltage drops and no armature flux to cause armature reaction. This change in the terminal voltage is significant in defining the voltage regulation.

The voltage regulation of an alternator is defined as the change in its terminal voltage when full load is removed, keeping field exicitation and speed constant, divided by the rated terminal voltage.

So if, V_{ph} = Rated thermal voltage

E_{ph} = No load induced e.m.f.

Then voltage regulation is defined as,

$$\% \text{ Reg} = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100$$

The value of the regulation not only depends on the load current but also on the power factor of the load. For lagging and unity p.f. conditions there is always drop in the terminal voltage hence regulation values are always positive. While for leading capacitive load conditions , the terminal voltage increases as load current Increases. Hence regulation is negative in such cases. The relationship between load current and the terminal voltage is called load characteristics for various load power factor conditions are shown in the fig. 38.

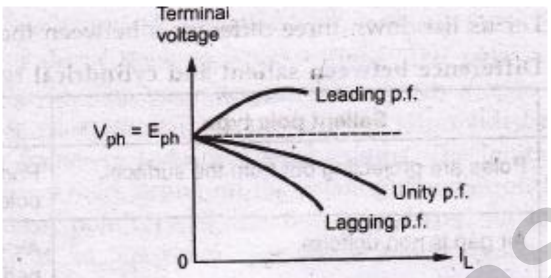


Fig. 38 Load characteristics of an alternator

c) A 3 phase, 6 pole, star connected alternator has 48 slots and 12 conductors per slot on the armature. If the rotor at 1200RPM and the flux per pole is 0.3 Wb, calculate the e.m.f. induced in the armature. The coils are full pitched and the winding factor is 0.95.

sol. : P = 6, slots = 48, conductors/ slots = 12, Φ = 0.3 wb

$$N_s = \frac{120\times f}{P}; \quad N_s = 1200 \text{ rpm}$$

$$f = \frac{PN_s}{120} = \frac{6\times1200}{120} = 60 \text{ Hz}$$

for full pitched winding, $K_c = 1$, $K_d = 0.95$

Total conductors = slots × conductors/slot = 48×12 = 576

$$Z_{ph} = \frac{Z}{3} = \frac{576}{3} = 192$$

$$T_{ph} = \frac{Z_{ph}}{2} = \frac{192}{2} = 96$$

$$E_{ph} = 4.44\, K_c\, K_d\, f\, \Phi T_{ph}$$

$$E_{ph} = 4.44\times1\times0.95\times60\times0.3\times96$$

$$= 75.9 \times 96$$

$$E_{ph} = 7288.70 \text{ volts}$$

$$E_{line} = \sqrt{3}\, E_{ph} = \sqrt{3}\times7288.70$$

$$E_{line} = 12.624 \text{ Kv}$$

Three Phase Induction Motors: Concept of rotating magnetic field. Principle of operation. Types and Constructional features. Slip and its significance. Applications of squirrel - cage and slip - ring motors. Necessity of a starter, star-delta starter. Illustrative examples on slip calculations.

UNIT-8

THREE PHASE INDUCTION MOTOR

In this chapter you will be introduced to asynchronous motors or induction motors.

- **Construction of squirrel cage and slip ring Induction motors**
- **Production of rotating flux**
- **Principle of operation**
- **Necessity of a starter for 3-phase induction motor**
- **Star –Delta starter**
- **Slip**

By the end of the chapter you will be able to answer questions like:

- **Why 3-phase induction motor cannot run with zero slip?**
- **Why 1-phase induction is motors not self starting?**
- **Why induction motors are the most preferred ac motors for many industrial applications?**
- **What do you understand by revolving flux?**

❖ INTRODUCTION

The asynchronous motors or the induction motors are most widely used ac motors in industry. They convert electrical energy in AC form into mechanical energy. They work on the principle of electromagnetic induction. They are simple and rugged in construction, quite economical with good operating characteristics and efficiency, requiring minimum maintenance, but have a low starting torque. They run at practically constant speed from no load to full load condition. The 3 - phase induction motors are self starting while the single phase motors are not self starting as they produce equal and opposite torques (zero resultant torque) making the rotor stationary. The speed of the squirrel cage induction motor cannot be varied easily.

❖ CLASSIFICATION -

They are basically classified into two types based on the rotor construction

1. Squirrel cage motor
2. Slip ring motor or phase wound motor

❖ CONSTRUCTION

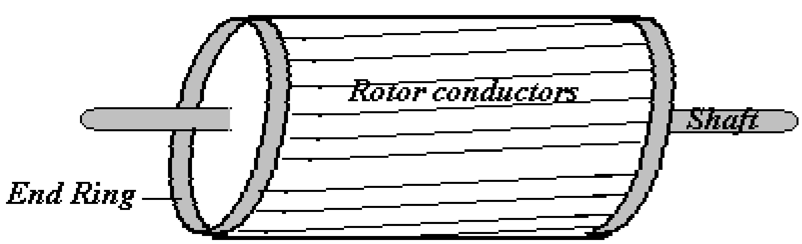
Three phase induction motor consists of two parts
(1) stator (2) rotor

➢ Stator

It is the stationary part of the motor supporting the entire motor assembly. This outer frame is made up of a single piece of cast iron in case of small machines. In case of larger machines they are fabricated in sections of steel and bolted together. The core is made of thin laminations of silicon steel and flash enameled to reduce eddy current and hysteresis losses. Slots are evenly spaced on the inner periphery of the laminations. Conductors insulated from each other are placed in these slots and are connected to form a balanced 3 - phase star or delta connected stator circuit. Depending on the desired speed the stator winding is wound for the required number of poles. Greater the speed lesser is the number of poles.

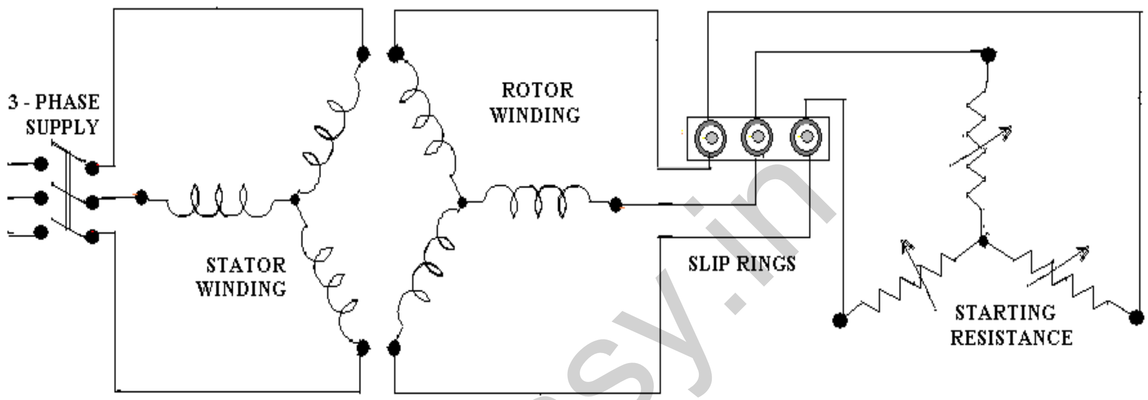
➢ Rotor

Squirrel cage rotors are widely used because of their ruggedness. The rotor consists of hollow laminated core with parallel slots provided on the outer periphery. The rotor conductors are solid bars of copper, aluminum or their alloys. The bars are inserted from the ends into the semi-enclosed slots and are brazed to the thick short circuited end rings. This sort of construction resembles a squirrel cage hence the name "squirrel cage induction motor". The rotor conductors being permanently short circuited prevent the addition of any external resistance to the rotor circuit to improve the inherent low starting torque. The rotor bars are not placed parallel to each other but are slightly skewed which reduces the magnetic hum and prevents cogging of the rotor and the stator teeth.



Squirrel cage induction rotor

The rotor in case of a phase wound/ slip ring motor has a 3-phase double layer distributed winding made up of coils, similar to that of an alternator. The rotor winding is usually star connected and is wound to the number of stator poles. The terminals are brought out and connected to three slip rings mounted on the rotor shaft with the brushes resting on the slip rings. The brushes are externally connected to the star connected rheostat in case a higher starting torque and modification in the speed torque characteristics are required. Under normal running conditions all the slip rings are automatically short circuited by a metal collar provided on the shaft and the condition is similar to that of a cage rotor. Provision is made to lift the brushes to reduce the frictional losses. The slip ring and the enclosures are made of phosphor bronze.



SLIP RING INDUCTION MOTOR

In both the type of motors the shaft and bearings (ball and roller) are designed for trouble free operation. Fans are provided on the shaft for effective circulation of air. The insulated (mica and varnish) stator and rotor windings are rigidly braced to withstand the short circuit forces and heavy centrifugal forces respectively. Care is taken to maintain a uniform air gap between the stator and the rotor.

- **Comparison of the squirrel cage and slip ring rotors**

The cage rotor has the following advantages:

1. Rugged in construction and economical.
2. Has a slightly higher efficiency and better power factor than slip ring motor.
3. The absence of slip rings and brushes eliminate the risk of sparking which helps in a totally enclosed fan cooled (TEFC) construction.

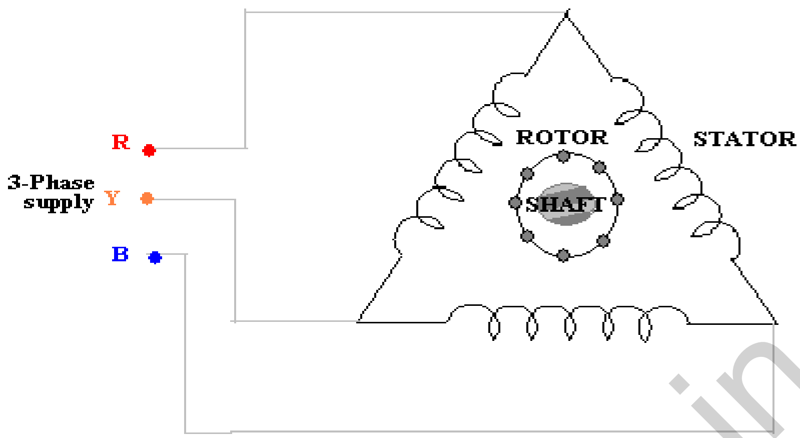
The advantages of the slip ring rotor are:

1. The starting torque is much higher and the starting current much lower when compared to a cage motor with the inclusion of external resistance.
2. The speed can be varied by means of solid state switching

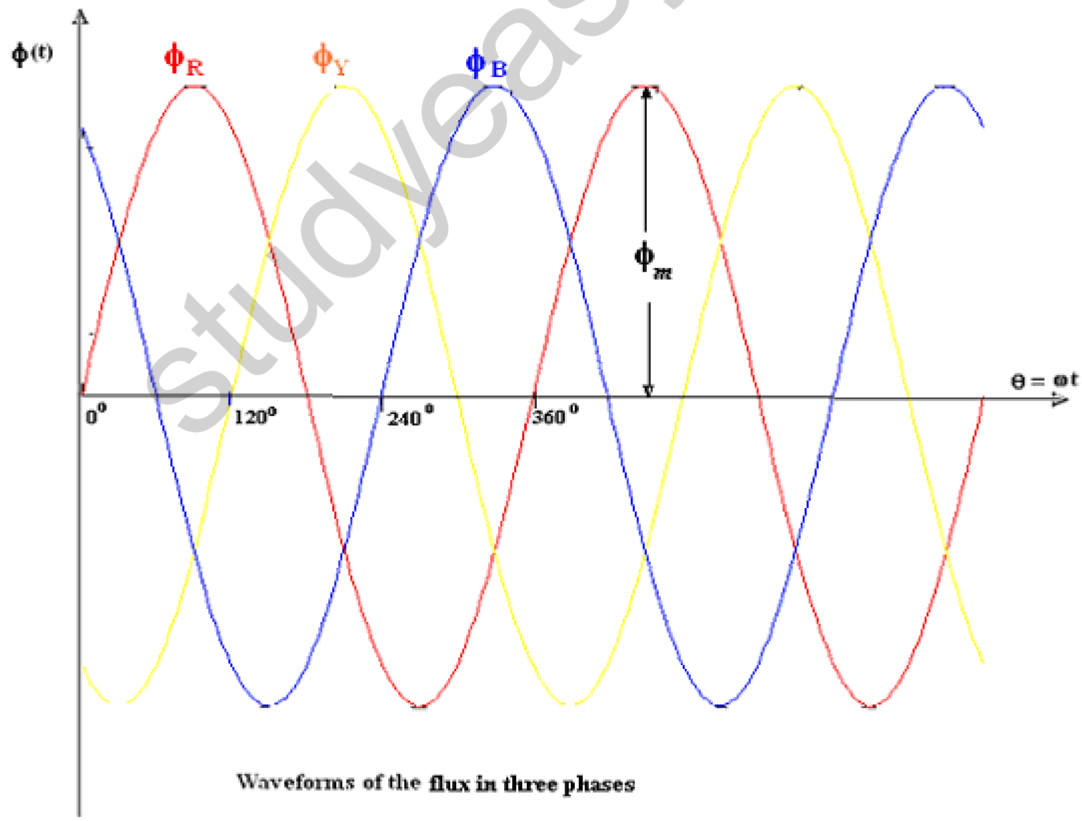
WORKING OF THE INDUCTION MOTOR

(a) Production of a rotating magnetic field

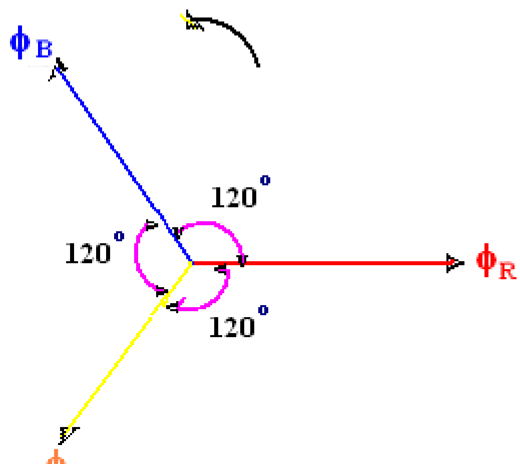
Consider a 3- phase induction motor whose stator windings mutually displaced from each other by 120° are connected in delta and energized by a 3- phase supply.



. The currents flowing in each phase will set up a flux in the respective phases as shown.



Waveforms of the flux in three phases



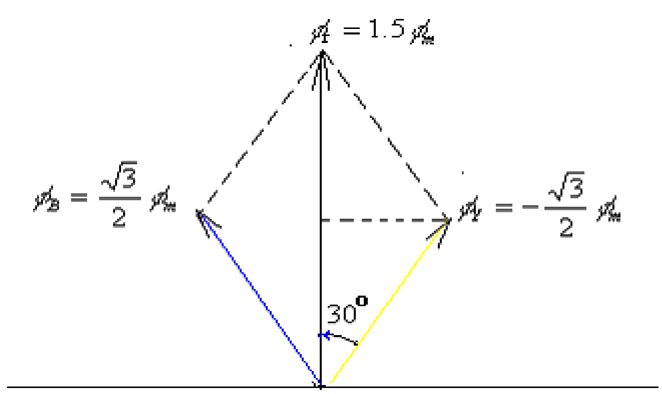
Phase Sequence RYB - Positive

The corresponding phase fluxes can be represented by the following equations

$\Phi_R = \Phi_m \sin \omega t = \Phi_m \sin \theta$
$\Phi_Y = \Phi_m \sin (\omega t - 120°)$
$\Phi_Y = \Phi_m \sin (\theta - 120°)$
$\Phi_B = \Phi_m \sin (\omega t - 240°)$
$\Phi_B = \Phi_m \sin (\theta - 240°)$

The resultant flux at any instant is given by the vector sum of the flux in each of the phases.

(i) When $\theta = 0^{\circ}$, from the flux waveform diagram ,we have



Resultant flux at θ = 0°

$\phi_R = 0$

$\phi_Y = \phi_{km} \sin(-120^\circ) = -\frac{\sqrt{3}}{2}\phi_m$

$\phi_B = \phi_m \sin(-240^\circ) = \frac{\sqrt{3}}{2}\phi_m$

The resultant flux ϕ_r is given by,

$\phi_r = 2 * \frac{\sqrt{3}}{2}\phi_m \cos(30^\circ) = 1..5\phi_m$

$\phi_B = \frac{\sqrt{3}}{2}\phi_m$

$\phi_Y = -\frac{\sqrt{3}}{2}\phi_m$

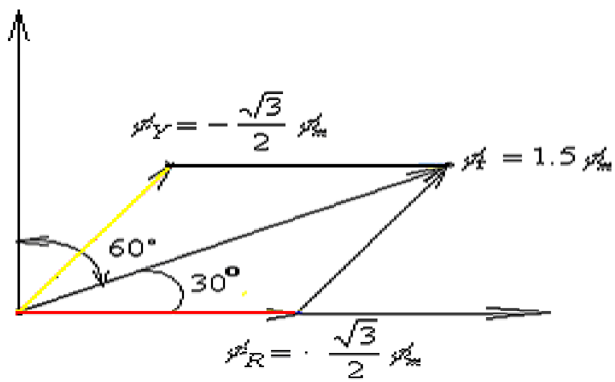
$\phi_r = 1.5\phi_m$

(ii) *When* $\theta = 60^\circ$

$\phi_R = \frac{\sqrt{3}}{2}\phi_m$

$\phi_Y = -\frac{\sqrt{3}}{2}\phi_m$

$\phi_B = 0$



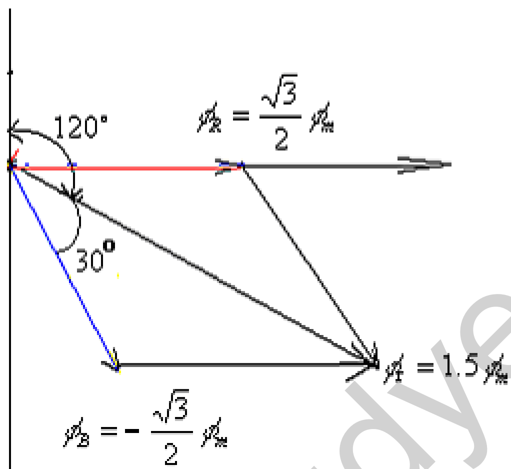
Resultant flux at θ = 60°

(iii) *When* $\theta = 120^\circ$

$$\phi_R = \frac{\sqrt{3}}{2}\phi_m$$

$$\phi_Y = 0$$

$$\phi_B = -\frac{\sqrt{3}}{2}\phi_m$$



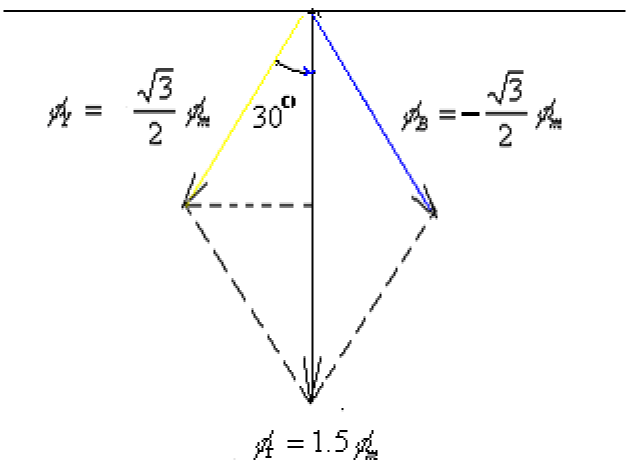
Resultant flux at $\theta = 120^\circ$

(iv) *When* $\theta = 180^\circ$

$$\phi_R = 0;$$

$$\phi_Y = \frac{\sqrt{3}}{2}\phi_m$$

$$\phi_B = -\frac{\sqrt{3}}{2}\phi$$



Resultant flux at θ = 180°

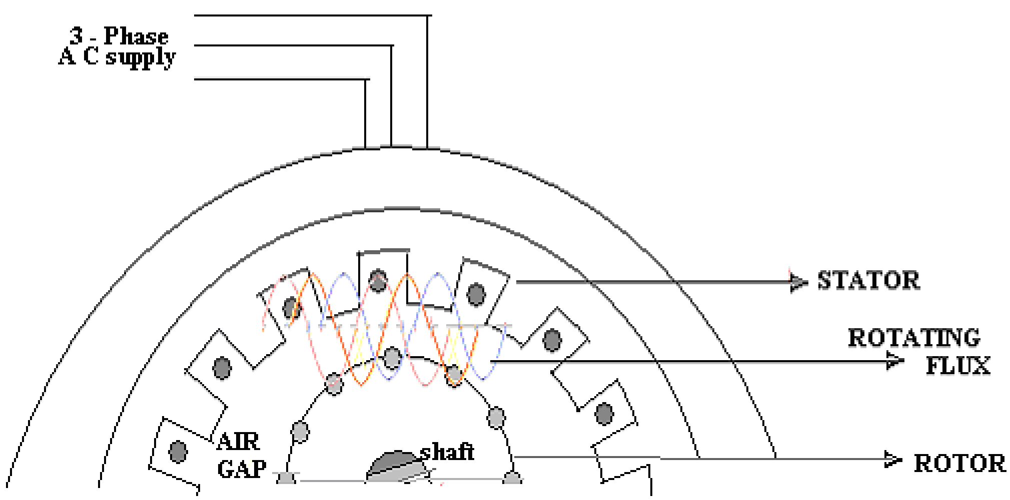
From the above discussion it is very clear that when the stator of a 3-phase induction motor is energized, a magnetic field of constant magnitude (1.5 φ_m) rotating at synchronous speed
(N_s) with respect to stator winding is produced.

(b) Rotation of the rotor

Consider a 3- phase stator winding energized from a 3 phase supply. As explained earlier a rotating magnetic field is produced running at a synchronous speed N_S

$$N_S = \frac{120\,f}{P}$$

Where f = supply frequency
P = Number of stator poles



ANIMATION INSTRUCTION

Consider a portion of 3- phase induction motor as shown in the above figure which is representative in nature. The rotating field crosses the air gap and cuts the initially stationary rotor conductors. Due to the relative speed between the rotating magnetic field and the initially stationary rotor,(change of flux linking with the conductor) an e.m.f. is induced in the rotor conductors, in accordance with the Faraday's laws of electromagnetic induction. Current flows in the rotor conductors as the rotor circuit is short circuited. Now the situation is similar to that of a current carrying conductor placed in a magnetic field. Hence, the rotor conductors experience a mechanical force which eventually leads to production of torque. This torque tends to move the rotor in the same direction as that of the rotating magnetic field.

❖ CONCEPT OF SLIP (*S*)

According to Lenz's law, the direction of rotor current will be such that they tend to oppose the cause producing it. The cause producing the rotor current is the relative speed between the rotating field and the stationary rotor. Hence, to reduce this relative speed, the rotor starts running in the same direction as that of stator field and tries to catch it. In practice the rotor can never reach the speed of the rotating magnetic field produced by the stator. This is because if rotor speed equals the synchronous speed, then there is no relative speed between the rotating magnetic field and the rotor. This makes the rotor current zero and hence no torque is produced and the rotor will tend to remain stationary. In practice, windage and friction losses cause the rotor to slow down. Hence, the rotor speed (N) is always less than the stator field speed (N_S). Thus the induction motor cannot run with **ZERO SLIP. The frequency of the rotor current** **$f_r = sf$.** The difference between the synchronous speed (N_S) of the rotating stator field and the actual rotor speed (N) is called the **slip speed.**

Slip speed = $N_S - N$ depends upon the load on the motor

$$\% \text{ Slip (s)} = \frac{N_S - N}{N_S} * 100$$

Note: In an induction motor the slip value ranges from 2% to 4%

APPLICATIONS OF INDUCTION MOTORS

Squirrel cage induction motor

Squirrel cage induction motors are simple and rugged in construction, are relatively cheap and require little maintenance. Hence, squirrel cage induction motors are preferred in most of the industrial applications such as in

i) Lathes
ii) Drilling machines
iii) Agricultural and industrial pumps
iv) Industrial drives.

Slip ring induction motors

Slip ring induction motors when compared to squirrel cage motors have high starting torque, smooth acceleration under heavy loads, adjustable speed and good running characteristics.

They are used in

i) Lifts
ii) Cranes
iii) Conveyors , etc.,

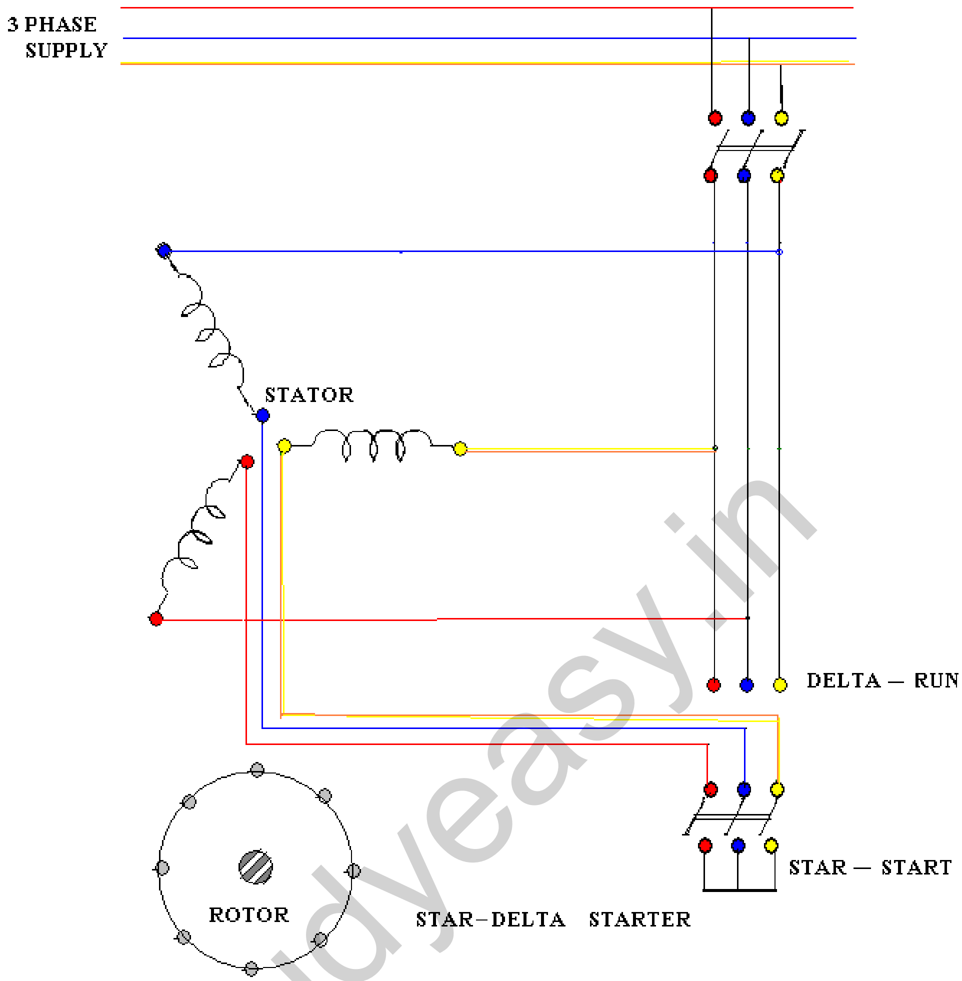
Necessity of starters for 3 phase induction motor

When a 3- phase motor of higher rating is switched on directly from the mains it draws a starting current of about 4 -7 times the full load (depending upon on the design) current. This will cause a drop in the voltage affecting the performance of other loads connected to the mains. Hence starters are used to limit the initial current drawn by the 3 phase induction motors.
The starting current is limited by applying reduced voltage in case of squirrel cage type induction motor and by increasing the impedance of the motor circuit in case of slip ring type induction motor. This can be achieved by the following methods.

1. Star –delta starter
2. Auto transformer starter
3. Soft starter

- **Star delta starter**

The star delta starter is used for squirrel cage induction motor whose stator winding is delta connected during normal running conditions. The two ends of each phase of the stator winding are drawn out and connected to the starter terminals as shown in the following figure.



When the switch is closed on the star-start side

(1) The winding is to be shown connected in star

(2) The current I = 1/3 * (I $_{\text{direct switching}}$)

(3) Reduction in voltage by1/√3

$$V = V_{supply} * 1/\sqrt{3}$$

When the switch is closed on to delta –run side

(1) the winding to be shown connected in delta

(2) application of normal voltage V $_{supply}$

(3) normal current I

During staring the starter switch is thrown on to the **STAR - START**. In this position the stator winding is connected in star fashion and the voltage per phase is **1/√3** of the supply voltage. This will limit the current at starting to **1/3** of the value drawn during direct switching. When the motor accelerates the starter switch is thrown on to the **DELTA - RUN** side. In this position the stator winding gets connected in the Δ fashion and the motor draws the normal rated current.

WORKED EXAMPLES

1. A 12 pole, 3 phase alternator is coupled to an engine running at 500 rpm. It supplies an Induction Motor which ahs a full load speed of 1440 rpm. Find the percentage slop and the number of poles of the motor.

Solution: N_A = synchronous speed of the alternator

$$F = \frac{PN_A}{120} = \frac{12 \times 500}{120} = 50 \text{ Hz (from alternator data)}$$

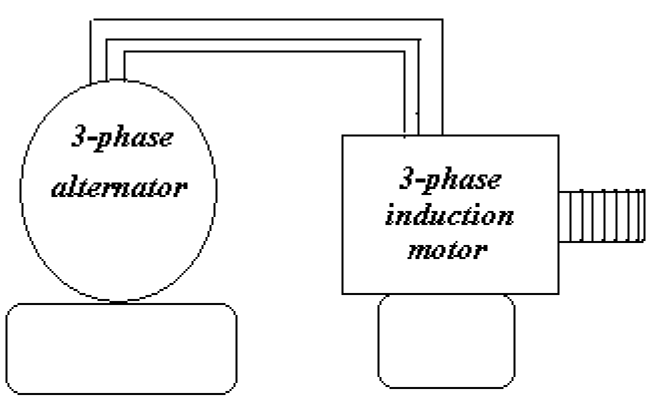
When the supply frequency is 50 Hz, the synchronous speed can be 750 rpm, 1500 rpm, 3000rpm etc., since the actual speed is 1440 rpm and the slip is always less than 5% the synchronous speed of the Induction motor is 1500 rpm.

$$s = \frac{N_S - N}{N_S} = \frac{1500 - 1440}{1500} = 0.04 \text{ OR } 4\%$$

$$N_S = \frac{120f}{P} = \frac{120 \times 50}{P} = 1500$$

$\therefore$ P = 4

2. A 6 pole induction motor is supplied by a 10 pole alternator, which is driven at 600 rpm. If the induction motor is running at 970 rpm, determine its percentage slip.



From alternator date: $f = \dfrac{P\,N_A}{120} = \dfrac{10 \times 600}{120} = 50$ Hz

Synchronous speed of the induction motor

From I.M. data: $N_S = \dfrac{120f}{P} = \dfrac{120 \times 50}{6} = 1000rpm$

$$\%slip = \frac{N_S - N}{N_S} \times 100 = \frac{1000 - 970}{1000} = 3\%$$

3. A 12 pole, 3 phase alternator is driven by a 440V, 3 phase, 6 pole Induction Motor running at a slip of 3%. Find frequency of the EMF generated by the alternator

For induction motor: $N_S = \dfrac{120f}{P} = \dfrac{120 \times 50}{6} = 1000rpm$

$$N = (1 - s)N_S = (1 - 0.03)1000 = 970rpm$$

As the alternator is driven by the Induction motor, the alternator runs at 970 r.p.m.

For alternator: $f = \dfrac{PN}{120} = \dfrac{12 \times 970}{120} = 97Hz$

4. A three phase 4 pole, 440 V, 50Hz induction motor runs with a slip of 4%. Find the rotor speed and frequency of the rotor current.

$$N_S = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500rpm$$

Solution: $S = \dfrac{N_S - N}{N_S}\; i.e. 0.04 = \dfrac{1500 - N}{1500}, \therefore N = 1440rpm$

$$f_r = sf = 0.04 \times 50 = 2Hz$$

5. A 3 phase, 50Hz 6 pole induction motor has a full load percentage slip of 3%.

Find(i) Synchronous speed and (ii) Actual Speed

$$= \frac{}{} = \frac{\times}{} =$$

$$= \frac{-}{} \qquad = \frac{-}{} \therefore \quad =$$

6. A 3 phase induction motor has 6 poles and runs at 960 RPM on full load. It is supplied from an alternator having 4 poles and running at 1500 RPM. Calculate the full load slip and the frequency of the rotor currents of the induction motor.

Solution:

$$f = \frac{PN}{120} = \frac{4 \times 1500}{120} = 50Hz(\textit{from alternator data})$$

forInduction motor

$$N_S = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000rpm$$

$$S = \frac{N_S - N}{N_S} = \frac{1000 - 960}{1000} = 0.04 \text{ or } 4\%$$

$$\text{fr} = sf = 0.04 \times 50 = 2Hz$$

7. The frequency of the e.m.f in the stator of a 4-pole induction motor is 50 Hz and that of the rotor is $1\frac{1}{2}Hz$. What is the slip and at what speed is the motor running?

Solution:

Given $P = 4$

$f = 50\text{Hz}$

$f_r = 1.5Hz$

- **To calculate slip (s)**

$$f_r = sf$$

$$1.5 = S \times 50$$

We have $s = \frac{1.5}{50}$

$$s = 0.03$$

$$s = 3\%$$

ii) To calculate the speed of the motor (*N*)

We have

$$N_s = \frac{120f}{P}$$

$$N_s = \frac{120 \times 50}{4} = 1500rpm$$

We also have

$$N = N_s(1 - s)$$
$$N = 1500(1 - 0.03)$$
$$N = 1455 rpm$$

8. A 3-phase, 60Hz induction motor has a slip of 3% at full load. Find the synchronous speed, the full- load speed and the frequency of rotor current at full load.

Solution

Given $P = 6$

$f = 60\text{Hz}$

$s = 3\% = 0.03$

- **To find the synchronous speed (N_s)**

We have

$$N_s = \frac{120f}{P} = \frac{120 \times 60}{6}$$
$$N_s = 1200 rpm$$

- **To calculate the full load speed (N)**

We have

$$N = N_s(1 - s) = 1200(1 - 0.03)$$
$$N = 1164 rpm$$

- **To calculate the frequency of the rotor current (f_r)**

We have

$$f_r = sf = 0.03 \times 60$$
$$f_r = 1.8 Hz$$

9. A 6-pole alternator running at 600 rpm. supplies a 3-phase, 4-pole induction motor. If the induction motor induced e.m.f makes 2 alternations per second, find the speed of the motor.

Solution

- **Alternator**

$P = 6$

$N_s = 600$

We have the frequency of induced e.m.f of an alternator given by

$$f = \frac{PN_s}{120} = \frac{6 \times 600}{120} = 30 Hz$$

Hence, induction motor receives the supply at 30Hz frequency

- **Induction motor**

It is given that the rotor induced e.m.f makes two alternations per second i.e. 1.0 cycle per second

$$\therefore f_r = 1.0Hz$$

We have

$$f_r = sf$$

$$s = \frac{f_r}{f} = \frac{1.0}{30}$$

$$s = 0.033$$

The speed of the rotating magnetic field is given by

$$N_s = \frac{120f}{P_m}$$ Where P_m = Number of poles of induction motor

$$N_s = \frac{120 \times 30}{4} = 900rpm$$

The speed of the induction motor (N) is given by

$$N = N_s(1 - s) = 900(1 - 0.033)$$

$$N = 870rpm$$

10. A 10-pole induction motor is supplied by a 6-pole alternator, which is driven at 1200rpm. If the motor runs with a slip of 3, what is the speed of the induction motor?

Solution

- **Alternator**

P = 6 pole

N_s = 1200rpm

Therefore, the frequency of the e.m.f generated is given by

$$f = \frac{PN_s}{120} = \frac{6 \times 1200}{120}$$

$$f = 60Hz$$

Hence, the induction motor is supplied at 60Hz frequency.

- **Induction motor**

Supply frequency = f = 60Hz

Slip = S = 3%

Stator poles = P_m =10

Speed of the rotating magnetic field,

$$N_s = \frac{120f}{P_m} = \frac{120 \times 60}{10}$$
$$N_s = 720rpm$$

Speed of the motor

$$N = N_s(-S) = 720(-0.03)$$
$$N = 698.4rpm$$

11. A 3-phase induction motor has 6-poles and runs at 960r.p.m. on full load. It is supplied from an alternator having 4-poles and running at 1500 r.p.m. calculate the full load slip of the motor.

Solution

- **Alternator**

P = 4 poles

N_s = 1500 rpm

The frequency generated e.m.f is given by

$$f = \frac{PN_s}{120} = \frac{4 \times 1500}{120}$$
$$f = 50Hz$$

Hence, the induction motor is supplied at 50 Hz

- **Induction motor**

$$P_m = 6\,poles$$
$$f = 50Hz$$
$$N = 960rpm$$

The speed of the rotating magnetic field is given by

$$= \frac{}{} = \frac{\times}{}$$

$$=$$

We have slip of an induction motor given by

$$S = \frac{N_s - N}{N_s} = \frac{1000 - 960}{1000} = 0.04$$
$$S = 4\%$$

12. A 4-pole, 30hp, 3-phase 400 volts, 50Hz induction motor operates at an efficiency of 0.85 with a power factor of 0.75(lag). Calculate the current drawn by the induction motor from the mains

Solution

Given P = 4
V = 400
ή = 0.85
cos φ = 0.75
Output= 30 hp
= 22.06538 Kw (1metric hp=735.5 watts)

We have,

$\eta = $ ———

= ——— = ———
η

=

But, for a 3-phase induction motor circuit, the power input is also given by the expression

$$P = \sqrt{3} V_L I_L \cos\Phi$$

$$I_L = \frac{P}{\sqrt{3} V_L \cos\Phi} = \frac{25.96 \times 1000}{\sqrt{3} \times 400 \times 0.75}$$

$$I_L = 49.94\, Amperes$$

13. A 5 hp, 400V, 50Hz, 6-pole, 3-phase induction motor operating on full load draws a line current of 7 amperes at 0.866 power factor with 2%slip. Find the rotor speed.

Solution.

Given *P* = 6
s = 2%
= 0.02
cos ϕ
= 0.866
f = 50 Hz = = ×
=
=

To find the rotor speed:
Speed of the rotor magnetic field is given by

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6}$$

$$N_s = 1000 rpm$$

Speed of the rotor = $N = N_s(1-s)$

$= 1000(1-0.02)$

$N = 980 rpm$

14. A 3-phase, 6-pole, 50 Hz induction motor has a slip of 1% at no-load and 3%at full-load. Find:

1. Synchronous speed,
2. No-load speed
3. Full-load speed,
4. Frequency of rotor current at standstill, and
5. Frequency of rotor current at full-load

Solution. Number of poles, p = 6

No- load slip, $s_0 = 1\%$

Full-load slip, $s_f = 3\%$

1. Synchronous speed,

$$N_s = \frac{120f}{p} = \frac{120 \times 50}{60} = 1000 r.p.m.(Ans)$$

2. No-load speed N_0,

We know that

$$s = \frac{N_s - N}{N_s} or N = N_s(1 - s)$$

$$N_0 = N_s(1 - s_0) = 100\left[1 - \frac{1}{100}\right] = 990 r.p.m.(Ans)$$

3. Full – load speed

$$N_f = N_s(1 - s_f) = 1000\left[1 - \frac{3}{100}\right] = 970 r.p.m.(Ans)$$

4. Frequency of rotor current at standstill, f_r

At standstill,

$$s = 1$$

$$f_r = sf = 1 \times 50 = 50 Hz.(Ans)$$

5. Frequency of rotor current at full – load, f_r =?

$$f_r = s \times f = \frac{3}{100} \times 50 = 1.5 Hz.(Ans)$$

15. A 3-phase, 12-pole alternator is coupled to an engine running at 500 rpm. The alternator supplies an induction motor which has a full-load speed of 1455rpm. Find the slip and number of poles of the motor.

Solution.
Number of poles of the alternator, p_a =12
Speed of the engine, N_e = 500rpm
Full-load speed of the induction motor, N_m = 1455rpm
Slip, s =?
Number of poles of the induction motor, P_m =?
Supply frequency,

$$f = \frac{N_a p_a}{120} = \frac{500 \times 12}{120} = 50Hz$$

When the supply frequency is 50Hz, the synchronous speed can be 3000, 1500, 1000, 750 rpm etc. since the full-load speed is 1455rpm and the full-load slip is always less than 4%, the synchronous speed is1500rpm.
Slip,

$$s = \frac{N_s - N}{N_s} = \frac{1500 - 1455}{1500} = 0.03 or 3\% (Ans)$$

Also,

$$N_s = \frac{120 f}{P_m}$$

$$P_m = \frac{120 f}{N_s} = \frac{120 \times 50}{1500} = 4 poles$$

Hence**, number of motor poles = 4. (Ans)**

16. A 4- pole, 50 Hz induction motor at no-load (N_{NL}) has a slip of 2%. When operated at full load the slip increases to 3%. Find the change in speed of the motor from no-load to full load.

$$N_s = \frac{120 f}{P}$$

$$= \frac{120 \text{ x } 50}{4} = 1500 rpm$$

$$no - load \text{ speed } N_{NL} = N_S (1 - S_{NL})$$

$$= 1500(1 - 0.02) = 1470 rpm$$

$$full\ load\ speed\ N = N_S (1 - S_{FL})$$

$$= 1500(1 - 0.03)$$

$$= 1455 rpm$$

change in speed from no - load to full load

$$N_{NL} - N_{FL} = 1470 - 1455 = 15 rpm$$

❖ REVIEW QUESTIONS

(1) What do you mean by rotating magnetic field and explain the production of torque in a three phase induction motor?
(2) A three phase, 50 Hz 6 pole induction motor has a full load percentage slip o 3%.Find the synchronous speed and the actual speed.
(3) Explain how torque is produced in a 3-phase induction motor?
(4) Why a 3-phase induction motor cannot run with zero slip?
(5) What do you understand by slip?
(6) Why is a single phase induction motor not self starting?
(7) Why starters are necessary for starting a 3-phase induction motor?
(8) Explain with a neat diagram the working of a STAR-DELTA starter?
(9) Bring out the differences between a squirrel cage and a slip ring induction motor.
(10) Why is squirrel cage induction motor widely used for industrial applications?
(11) Mention the applications of squirrel cage and slip ring induction motors.

SOLUTION TO QUESTION PAPERS

What do you mean by rotating magnetic field and explain the production of torque in a three phase induction motor.

Sol. : Rotating magnetic field (R.M.F) :

The rotating magnetic field can be defined as the field or flux having constant amplitude but whose axis is continuously rotating in a plane with certain speed. So if the arrangement is made to rotate a permanent magnet, then the resulting field is a rotating magnetic field. But in this method it is necessary to rotate a magnet physically tp produce rotating magnetic field.

But in three phase induction motors such a rotating magnetic field is produced by supplying currents to a set of stationary windings, with the help of three phase a.c. supply. The current carrying windings produce the magnetic field or flux . And due to interaction of three fluxes produced due to three phase supply, resultant flux has a constant magnitude and its axis rotating the windings. This type of field is nothing, but rotating magnetic field. Let us study how it happens.

Production of RMF :

A three phase induction motor consists of three phase winding as its stationary part called stator. The three phase stator winding is connected in star or delta. The three phase windings are displaced from each other by 120^0. The windings are supplied by a balanced three phase a.c. supply. This is shown in the fig. 39. The three phase windings are denoted as R-R', Y-Y' and B- B'.

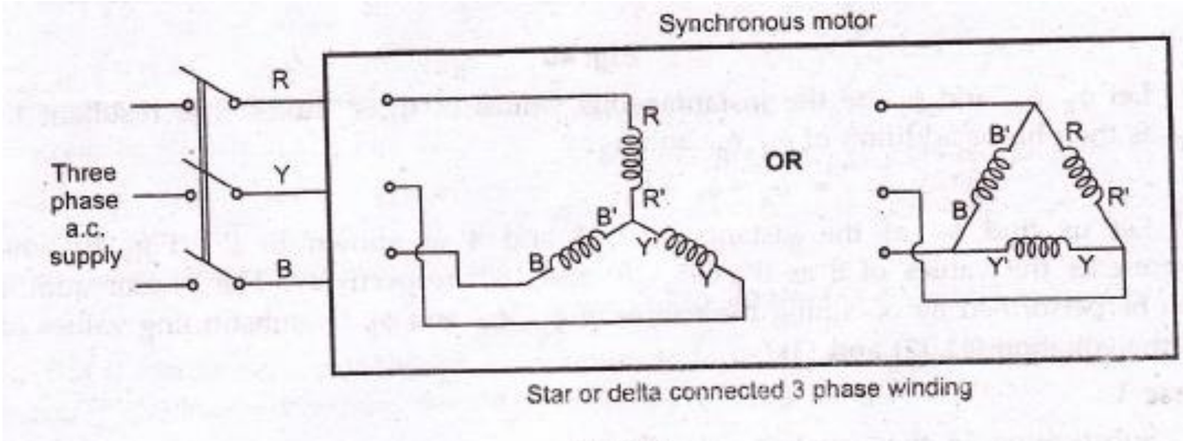


Fig. 39

The three phase currents flow simultaneously through the windings and are displaced from each other by 120^0 electrical. Each alternating phase current produces its own flux which is sinusoidal. So all three fluxes are sinusoidal and are seprated from each other by 120^0 . If the phase sequence of the windings is R –Y –B, then mathematical equations for the instantaneous values of the three fluxes Φ_R, Φ_Y and Φ_B can be written as,

$$\Phi_R = \Phi_m \sin(\omega t) = \Phi_m \sin\theta \quad \text{..... (1)}$$

$$\Phi_Y = \Phi_m \sin(\omega t-120^0) = \Phi_m \sin(\theta -120^0) \quad \text{....... (2)}$$

$$\Phi_B = \Phi_m \sin(\omega t-140^0) = \Phi_m \sin(\theta -240^0) \quad \text{........ (3)}$$

As windings are indentical and supply is balanced, the magnitude of each flux is Φ_m. due to phase sequence R- Y – B, flux Φ_Y lags behind Φ_R by 120^0 and Φ_B lags Φ_Y by 120^0. So Φ_B ultimately lags Φ_R by 240^0 . The flux Φ_R is talen as reference while writing the equations.

The fig. 40 shows the waveforms of three fluxes in space. The fig. 40 shows the phasor diagram which clearly shows the assumed positive directions of each flux assumed positive direction means whenever the flux is positive it must be represented along the direction shown and whenever the flux is negative it must be represented along the opposite direction to the assumed positive direction.

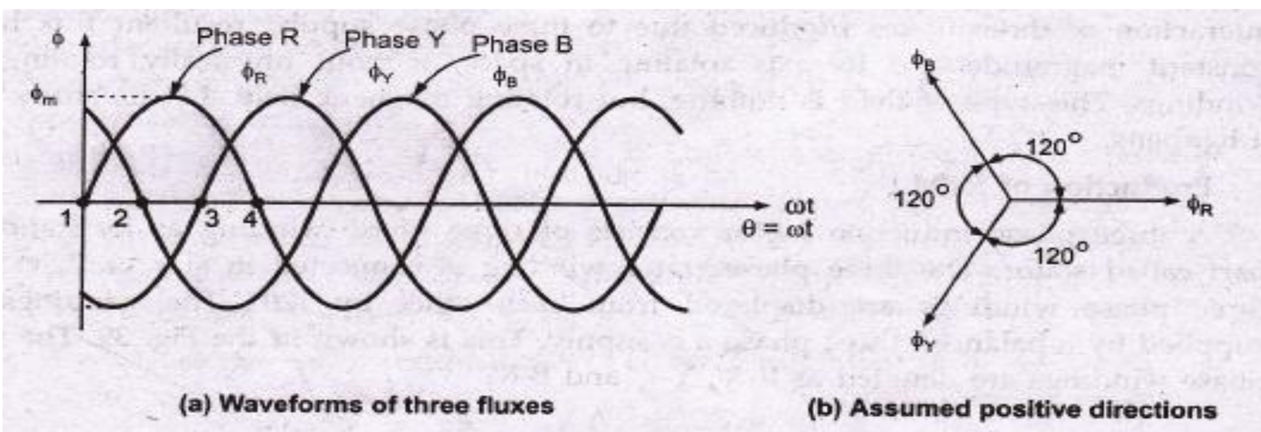


Fig. 40

Let ϕ_R, ϕ_Y and ϕ_B be the instantaneous values of three fluxes. The resultant flux ϕ_T as the phasor addition of $\overline{\phi_R}$, $\overline{\phi_Y}$ and $\overline{\phi_B}$.

$$\overline{\phi_T} = \overline{\phi_R} + \overline{\phi_Y} + \overline{\phi_B}$$

Let us find ϕ_T at the instants 1, 2, 3, and 4 as shown in the fig. 40 which represents the values of θ as 0^0 , 60^0, 120^0 and 180^0 respectively. The phasor addition can be performed by obtaining the values of ϕ_R, ϕ_Y and ϕ_B by substituting values of θ in the equation (1), (2) and (3).

Case 1:

Substituting in the equations (1), (2) and (3) we get,

$$\Phi_R = \Phi_m \sin 0^0 = 0$$
$$\Phi_Y = \Phi_m \sin(-120^0) = -0.866\Phi_m$$
$$\Phi_B = \Phi_m \sin(-240^0) = +0.866\Phi_m$$

The phasor addition is shown in the fig. 41. the positive values are shown in assumed positive directions while negative values are shown in opposite direction to the assumed positive direction of the respective fluxes. Refer to assumed positive directions shown in the fig. 41.

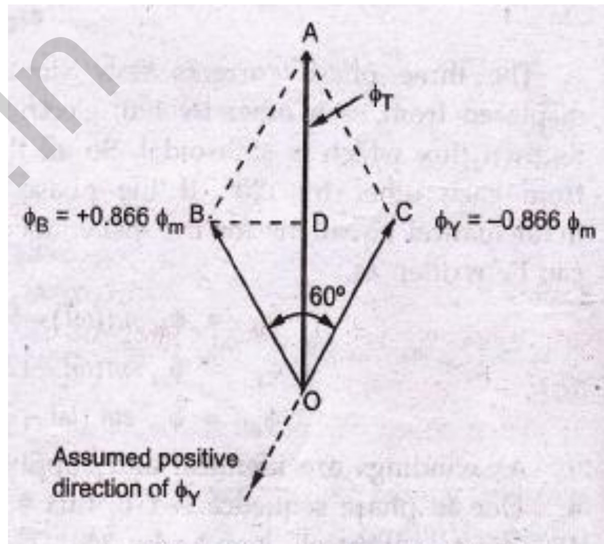


Fig. 41 Vector diagram for θ = 0°

BD is drawn perpendicular from B on ϕ_T . It bisects Φ_T.

$$OD = DA = \frac{\phi_T}{2}$$

In triangle OBD, ∠ BOD = 30^0

$$\text{Cos } 30^0 = \frac{OD}{OB} = \frac{\phi\ /2}{0}$$

$$\phi_T = 2\times 0.866\phi_m \times \cos 30^0$$
$$= 1.5\ \phi_m$$

So magnitude of ϕ_T is 1.5 ϕ_m and its position is vertically upwards at θ = 0^0.

Case 2 :

Equations (1), (2) and (3) give us,

$$\phi_R = \phi_m \sin 60^0 = +0.866\phi_m$$
$$\phi_Y = \phi_m \sin(-60^0) = -0.866\phi_m$$
$$\phi_B = \phi_m \sin(-180^0) = 0$$

So ϕ_R is positive and ϕ_Y is negative and hence drawing in appropriate directions we get phasor diagram as shown in the fig. 42.

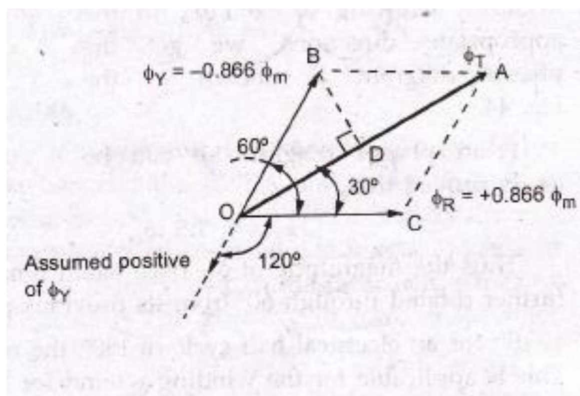


Fig. 42 Vector diagram for θ = 60°

Doing the same construction drawing perpendicular from B on ϕ_T at D. we get the same result as

$$\phi_T = 1.5\phi_m$$

But it can be seen that through its magnitude is 1.5 ϕ_m it has rotated through 60^0 in space, in clockwise direction, from its previous position.

Case 3:

Equations (1), (2) and (3) give us,

$_R = \ _m \sin 120 = + 0.866 \ _m$

$\phi_Y = \phi_m \sin 0 = 0$ ϕ

$\phi_B = \phi_m \sin (-120^0) = -0.866 \ _m$

So ϕ ϕ ϕ

ϕ_R is positive and ϕ_B is negative.

Showing ϕ_R and ϕ_B in the appropriate directions, we get the phasor dioagram as shown in the fig. 43.

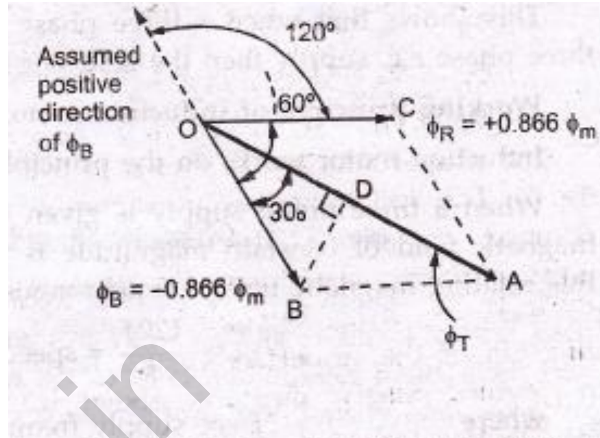


Fig. 43 Vector diagram for θ =120°

After doing the construction same as before i.e. drawing perpendicular from B on ϕ_T ,it can be proved again that,

$\phi_T = 1.5\phi_m$

But the position of ϕ_T is such that it has rotated further through 60^0 from its previous position, in clockwise direction. And from its position at $\theta = 0^0$, it has rotated through 120^0 in space, in clock wise direction.

Case 4: $\theta = 180^0$

Fropm equations (1), (2) and (3),

$\phi_R = \phi_m \sin (180^0) = 0$

$\phi_Y = \phi_m \sin (60^0) = + 0.866\phi_m$

$\phi_B = \phi_m \sin (-60^0) = - 0.866\phi_m$

So $\phi_R = 0$, ϕ_Y is positive and ϕ_B is negative.

Drawing ϕ_Y and ϕ_B in the appropriate directions, we get the phasor diagram as shown in the fig.44

From phasor, it can be easily proved that,

$\phi_T = 1.5 \ \phi_m$

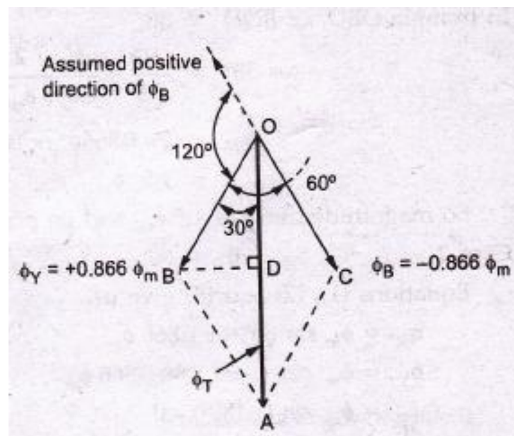


Fig. 44 Vector diagram for θ =180°

Thus the magnitude of ϕ_T once again remains same. But it can be seen that it has further rotated through 60^0 from its previous position in clock wise direction.

So for an electrical half cycle of 180^0, the resultantϕ_T has also rotated through 180^0. This is applicable for winding wound for 2 poles.

From the above discussion we have following conclusions :

a) The resultant of the three alternating fluxes, separated from each other by 120^0, has a constant amplitude of 1.5 ϕ_m where ϕ_m is maximum amplitude of an individual plux due to any phase.

b) The resulatant always keeps on rotating with a certain speed in space.

This shows that when a three phase stationary windings are exicited by balanced three phase a.c. supply then the resulting field produced is rotating magnetic field.

2) **Working principle of induction motor :**

Induction motor works on the principle of electro magnetic induction.

When a three phase supply is given to the three phase stator winding, a rotating magnetic field of constant magnitude is produced as discussed earlier. The speed of this rotating magnetic field is synchronous speed, Ns r.p.m

$N_S = \frac{120f}{P}$ = speed of rotating magnetic field

f = supply frequency

P = number of poles for which stator winding is wound.

This rotating field produces an effect of rotating poles around a rotor. let direction of rotation of this rotating magnetic field is clock wise as shown in the fig. 45.

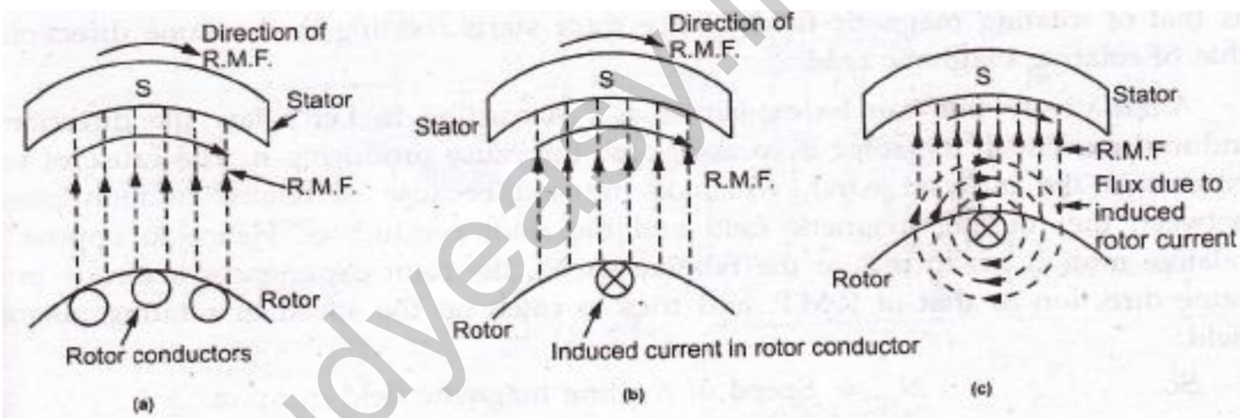


Fig. 45

Now at this instant rotor is stationary and stator flux R.M.F. is rotating. So its pbvious that thre exists a relative motion between the R.M.F. and rotor conductors. Now the R.M.F. gets cut by rotor conductor as R.M.F. sweeps over rotor conductors. whenever conductor cuts flux , e.m.f. gets induced init. so e.m.f. gets induced in the rotor conductors called rotor induced e.m.f. this is electro magnetic induction. As rotor forms closed circuit, induced e.m.f circulates current through rotor called rotor current as shown in fig. 45. let direction of this current is going into paper denoted by a cross as shown in the fig. 45.

Any current carrying conductor produces its own flux. so rotor produces its flux called rotor flux. for assumed direction of rotor current, the direction of rotor flux is clock wise as shown in the fig. 46. this direction can be easily determined using right hand thumb rule. now there are two fluxes, one R.M.F. and other rotor flux. Both the fluxes interact

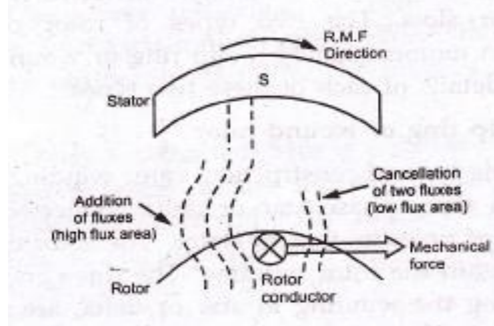


Fig. 46 Interaction of fluxes

with each asa shown in the fig. 46. on left
of rotpr conductor two fluxes are in same direction hence add up to get high flux area. on

right side, two fluxes cancel each other to produce low flux area. As flux lines act as stretched rubber band, high flux dendity area exerts a puch on rotor conductor towards low flux dendity area. so rotor conductor experiences a force from left to right in this case, as shown in the fig. 46, due to interaction of the two fluxes.

As all the rotor conductors experience a force, the overall rotor experiences a torque and starts rotating. so interaction of the two fluxes is very essential for a motoring action. As seen from the fig. 46, the direction of force experienced is same as that of rotating magnetic field. hence rotor starts rotating in the same direction as that of rotating magnetic field.

Alternatively this can be explained as : according to Len'Z law the direction of induced current in the rotor is so as oppose the cause producing it. The cause of rotor current is the induced e.m.f which is induced because of relative motion present between the rotating magnetic field and the rotor conductors. hence to oppose the relative motion i.e. to reduce the relative speed, the rotor experiences a torque in the same direction as that of R.M.F. and tries to catch up the speed of rotating magnetic field.

so, N_S = speed of rotating magnetic field in r.p.m

N = speed of rotor i.e. motor in r.p.m.

$N_S - N$ = Relative speed between the two rotating magnetic filed and the rotor conductors.

Thus rotor always rotates in same direction as that of R.M.F.

3)Discuss the important features and advantages of squirrel cage and phase wound rotor constructions in an induction motor.

sol. : Rotor constructions in induction motor :

The rotor is placed inside the stator. The rotor is also laminated in construction slotted at the periphery and cylindrical in shape. The rotor conductors are placed the rotor slots. The two types of rotor construction are generally used for the induction motors namely i) slip ring or wound rotor and ii) squirrel cage rotor. Let see the details of each of these two types:

i) slip ring or wound rotor :

In this type of construction, rotor winding is exactly similar to the stator. The rotor carries a three phase star or connected, disturbed winding, wound for same number of poles as that of stator. The rotor construction is laminated and slotted slots contain the rotor winding. The three ends of three phase winding, available after connecting the windings in star or delts, are permanently connected to the slip rings. The slip rings are mounted on the same shaft.

The slip rings are used to connect external stationary circuit to the internal rotating circuit. so in this type of rotor, the external resistances can be added with the help brushes and slip ring arrangement, in series with each phase of the rotor winding.

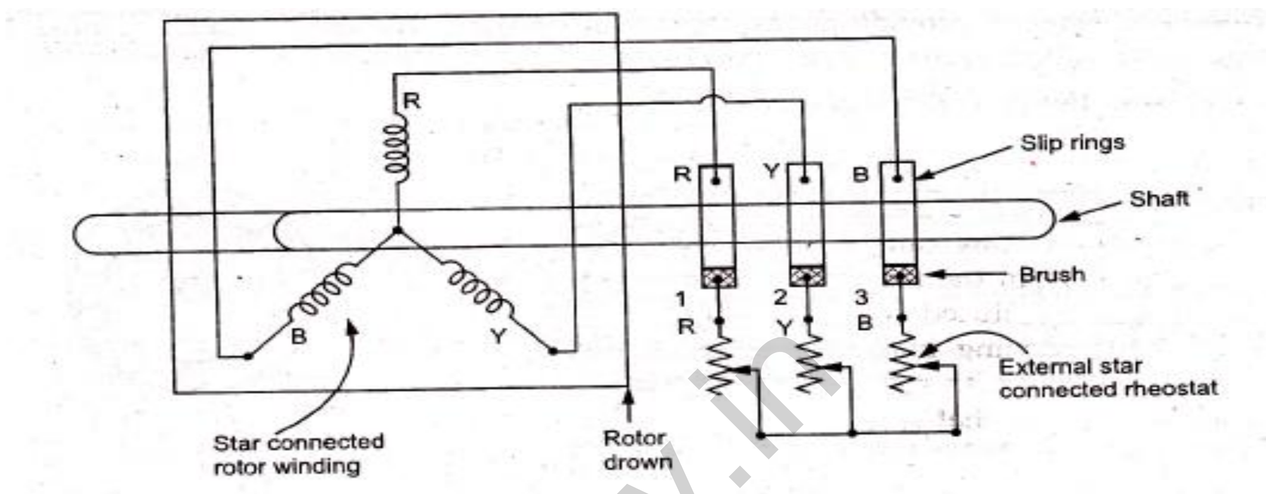


Fig. 47 Slip ring rotor or wound rotor

Action of slip rings and brushes :
The slip rings are mounted on the shaft. On end to each phase winding, after connecting the winding in star or delta is connected to the slip ring. Thus there are three slip rings. The three slip rings now behave as the terminals R,Y and B of rotor winding. Now as rotor rotates, slip rings also rotate. We want to add external resistance in series with rotor. But external resistance is stationary while rotor is rotating. Thus to have connection between rotating and stationary members, the brushes are used. The brushes are stationary and resting on slip rings to make electric contact with them. Thus slip rings are rotating terminals of rotor while brushes are stationary terminals of rotor. Now whatever resistance we connected across the brushes get internally added in series with each other winding phase, through rotor is rotating. This is the action of slip ring and brush assembly. The brushes are made up of carbon.

In the running condition, the slip rings are shorted. This is possible by connecting a metal collar which gets pushed and connects all the slip rings together, shorting them. At the same time brushes are also lifted from the slip rings. This avoids wear and tear of the brushes due to friction. The possibility of addition of an external resistance in series with the rotor, with the help of slip rings is the main feature of this type of rotor.
Such addition of resistance in rotor circuit only at start is used to improve starting torque of the motor.

3) **Squirrel cage rotor :**
When there is no need of controlling starting torque of the motor, the rotor construction can be made very simple. In this type, rotor consists of uninsulated copper or aluminium bars, placed in the slots. These bars are permanently shorted at each end with the help of conducting end ring. The bars are usually brazed to the end rings to provide good

mechanical strength. The entire structure looks like a cage hence called squirrel cage rotor.

This is shown in the fig. 48.

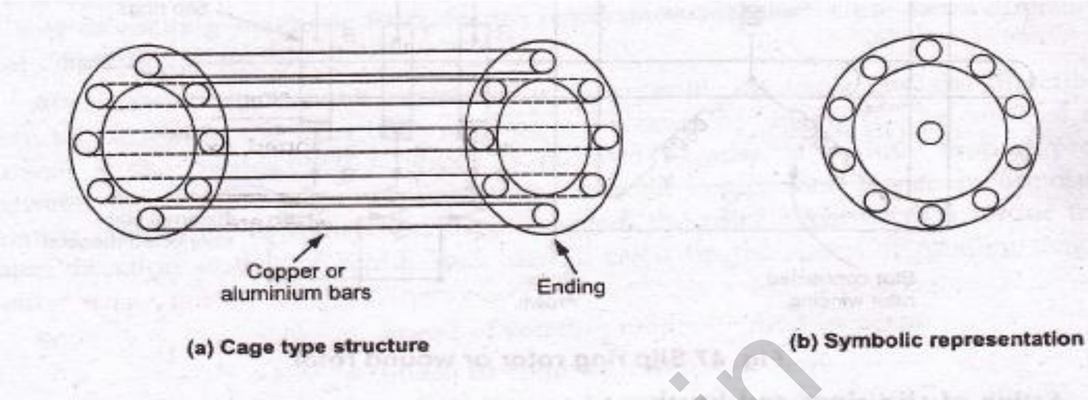


Fig. 48 Squirrel cage rotor

As the bars are permanently shorted to each other, the resistance of the entire rotor is very very small. hence rotor is also called short circuited rotor. As the rotor is shorted on its own, no external resistance can have any effect on the rotor resistance. Hence for this rotor, no external resistance can be added in the rotor circuit. So slip rings and brush assembly is absent in this type of rotor.

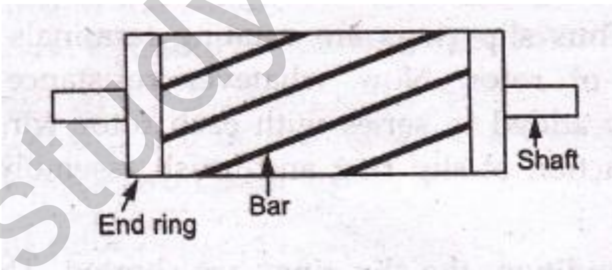


Fig. 49 Skewed bars

In practice, the bars are slightly skewed as shown in the fig. 49. The skewing helps in two ways :

1) It helps to reduce noise due to magnetic hum.
2) It reduces the tendency of magnetic locking between rotor and stator.

Fan blades are generally provided at the ends of the rotor core. This circuliates the air through the machine while operation, providing the necessary cooling. The air gap between stator and rotor is kept uniform and as small as possible.

The important points of comparison between the two types of rotor are given as follows.

5)Comparison of squirrel cage and wound rotor :

	Wound or slip ring rotor	Squirrel cage rotor
1	Rotor consists of a three phase winding similar to the stator winding.	Rotor consists of bars which are shorted at the ends with the help of end rings
2	Construction is complicated.	Construction is simple.
3	Resistance can be added externally	As permanently shorted,external resistance cannot be added.
4	Slip rings and brushes are present to add external resistance.	Slip rings and brushes are absent.
5	The construction is delicate and due to brushes, frequent maintenance is necessary.	The construction is robust and maintenance free.
6	The rotors are very costly.	Due to simple construction, the rotors are cheap.
7	Only 5% of induction motors in industry use slip ring rotor.	Very common and almost 95% induction motors use this type of rotor.
8	High starting torque can be obtained.	Moderate starting torque which cannot be controlled..
9	Rotor resistance starter can be used	Rotor resistance starter cannot be used
10	Rotor must be wound for the same number of poles as that of stator.	The rotor automatically adujusts itself for the same number of poles as that of stator.
11	Speed control by rotor resistance is possible.	Speed control by rotor resistance is not possible.
12	Rotor copper losses are high hence efficiency is less.	Rotor copper losses are less hence have higher efficiency is less.
13	Used for lifts, hoists, cranes, elevators, compressors etc.	Used for lathes, drilling machines , fans, blowers, water pumps, grinders, printing machines etc.

6) A 9 phase, 50 Hz, 6 pole induction motor has a full load percentage slip of 30%. Find i) synchronous speed and ii) Actual speed. **[4]**

Sol. :

f = 50 Hz

P = 6

Slip, S = 3% = 0.03

$$N_S = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$\text{Slip, } S = \frac{N_S - N}{N_S} \times 100$$

$$\frac{3}{100} = \frac{1000 - N}{1000}$$

(0.03) (1000) = 1000 – N

30 = 1000 – N

N = 1000 – 30 = 970 rpm

Rotor speed = 970 rpm

----- -----END----- ----